**Review for the First Mid-Term of Math 21b**
**October 24, 2007**

- Coefficient matrix and augmented matrix of a system of linear equations.

- Reduced row-echelon form of a matrix characterized by three conditions:

  *a.* If a row has nonzero entries, then the first nonzero entry is 1, called the leading 1 in this row.

  *b.* If a column contains a leading 1, then all other entries in that column are zero.

  *c.* If a row contains a leading 1, then each row above contains a leading 1 further to the left.

- Reduction of a matrix to reduced row-echelon form by using three kinds of row operations (swapping rows, multiplying a row by a nonzero number, and adding a multiple of a row to another).

- Use reduction to reduced row-echelon form to determine whether a system of linear equations is inconsistent, uniquely solvable, or solvable with an infinite number of solutions and to give a general solution (if it exists) by using free variables.

- Use reduction to reduced row-echelon form to determine whether a square matrix is invertible and to find its inverse if it is invertible.

- Determinant of a $2 \times 2$ matrix. Formula for the inverse matrix of a $2 \times 2$ matrix with nonzero determinant.

- The span of a set of vectors. Redundant vectors in a sequence of vectors. Linear dependence and independence of a set of vectors. Subspaces. Bases. Dimension.

- Determine the rank and the nullity of a matrix. Find a basis for the image and for the kernel of a matrix. Rank-Nullity Theorem.

- Equivalence conditions for the invertibility of an $n \times n$ matrix $A$: unique solvability of $A\vec{x} = \vec{b}$, $\mathrm{rref}(A) = I_n$, $\mathrm{rank}(A) = n$, $\mathrm{im}(A) = \mathbb{R}^n$, $\ker(A) = \{\vec{0}\}$, column vectors forming a basis of $\mathbb{R}^n$, column vectors spanning $\mathbb{R}^n$, column vectors linearly independent.

- Special linear transformations: rotations, dilations, projections (onto a line or a plane), reflections, and shears (horizontal and vertical).

- The column vectors of the matrix of a linear transformation equal to its images of the standard vectors.

- Relation between matrix multiplication and the composition of linear transformations.

- Coordinates with respect a basis of a subspace. Matrix of a linear transformation with respect to a basis. Relation of matrices of the same linear transformation with respect to two different bases. Similar matrices. Powers of similar matrices. Similarity as an equivalence relation.

- Concept a linear space (also known as a vector space). Addition and scalar multiplication in a linear space and the laws (associativity, commutativity, distributivity, etc.) satisfied by them. Examples of linear spaces: solutions of differential equations, spaces of polynomials, spaces of matrices, etc. Dimension of linear space. Finite and infinite dimension.

> The First Midterm covers up to and including Section 4.1 of Bretscher's book on Linear Algebra with Applications