Review for Second Midterm of Math 21b, November 28, 2007

(1) Gram-Schmidt process of constructing orthonormal vectors from linearly independent vectors.

(2) QR decomposition of an $n \times m$ matrix A in the form QR, where Q is an $n \times m$ matrix whose column vectors are orthonormal and R is an $m \times m$ matrix which is upper triangular with positive diagonal entries.

(3) Orthogonal transformations as length-preserving and orthogonality-preserving transformations. Orthonormal set of vectors and orthonormal basis. Pythagorean theorem. Cauchy-Schwarz inequality. Angle between vectors.

(4) Transpose of a matrix. Product of transposes of matrices and inverse of the transpose of a matrix. Symmetric and skew-symmetric matrices. Inner product of two vectors as the matrix product of the transpose of a vector and the other vector. The kernel of the transpose of a matrix as orthogonal complement of the image of the matrix.

(5) Formula for the orthogonal projection onto a subspace spanned by orthonormal vectors. Formula for the orthogonal projection onto a subspace spanned by linearly independent vectors. Formula for the least-squares solution of a (possibly inconsistent) system of linear equations.

(6) Determinant defined by induction and expansion down the first column. Determinant computed by expansion down any column and across any row. Formula for the determinant of a 2×2 matrix. Formula for the determinant of a 3×3 matrix. Formula for the determinant of an upper or lower triangular matrix. Formula for the determinant of an upper or lower triangular partitioned matrix with square matrices as diagonal entries.

(7) Effect of Gauss-Jordan row operations on a determinant. Computation of determinant by Gaussian elimination. Determinant of the transpose of a matrix. Determinant of a product of matrices. Determinant of the inverse of a matrix. Determinants of similar matrices.

(8) Characteristic equation of a square matrix. Trace of a matrix. Eigenvalues, eigenvectors, eigenspaces and their computations. Eigenbasis and diagonalization. Algebraic and geometric multiplicities and their relation.

Diagonalization of a matrix whose eigenvalues are all real and distinct. Eigenbasis and diagonalization for a linear transformation of a linear space (with domain and codomain of the linear transformation both equal to the same linear space).

(9) Discrete linear dynamical systems. Closed formula for the state vector of the system by using diagonalization with respect to an eigenbasis. Discrete trajectories and phase portraits.

(10) Complex eigenvalues. Any real 2×2 matrix with non-real eigenvalues is similar to a rotation-scaling matrix. Formula for such a similarity.

(11) For a dynamical system defined by a real 2×2 matrix the zero state is asymptotically stable if and only if the modulus of all the complex eigenvalues is less than 1.

> The Second Midterm covers up to and including Section 7.6 of Bretscher's book on Linear Algebra with Applications, except Sections 4.2, 4.3, 5.5, and 6.3.