Spring 98 Exam 1

1) Suppose each TV viewer watches exactly one show each night: either Winter Olympics, Nightline, or the Simpsons. Of Olympics viewers on a given night, 30% switch to Nightline the next night, and 10% to the Simpsons. Of Nightline viewers on a given night, 40% go to the Olympics and 10% to the Simpsons the next night. Of Simpsons' viewers, none switch to Nightline, but 40% switch to the Olympics the next night. Viewers who do not switch shows continue to watch the same show as the previous night. Surprisingly, the percentages (of total TV viewers) for each show stay constant. Find these percentages.

2) Let **A** be the matrix $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

- Describe geometrically the transformation of the plane given by **A** and draw a sketch of the effect of **A** on the face below. (A happy face with chin at (0,0) and forehead at (0,1).)
- Describe geometrically the linear transformation of the plane given by $\mathbf{A}^4$ and draw a sketch of the effect of $\mathbf{A}^4$ on the face below.
- Find the matrix $\mathbf{A}^4$.
- Find the matrix $\mathbf{A}^{17}$.

3) Let L be the line in $\mathbf{R}^3$ spanned by (1,1,-1). Let **A** denote the matrix for orthogonal projection onto L. Let **B** denote the matrix for reflection in L. Let **C** denote the matrix for rotation through 120 degrees around L, clockwise when facing the origin from (1,1,-1). Let **D** denote the matrix for rotation through 90 degrees about the y-axis (clockwise as seen when facing the origin from the positive y-axis.) Let **E** denote the composition of two rotations: first a clockwise rotation of 90 degrees around the z-axis, and then the rotation **D**. The following five matrices are the matrices **A** through **E** in some order. Identify them.

$$\begin{bmatrix} -1/3 & 2/3 & -2/3 \\ 2/3 & -1/3 & -2/3 \\ -2/3 & -2/3 & -1/3 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1/3 & 1/3 & -1/3 \\ 1/3 & 1/3 & -1/3 \\ -1/3 & -1/3 & 1/3 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

4) Find a basis for the kernel and a basis for the image of the linear transformation from $\mathbf{R}^4$ to $\mathbf{R}^3$ given by the matrix

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 2 & 2 \\ 1 & 2 & 1 & 2 \\ 2 & 4 & -1 & 1 \end{bmatrix}$$

5) In each part of this question, W is a subspace of $\mathbf{R}^3$ and V is a subspace of $\mathbf{R}^2$. In each part, either exhibit the matrix of the linear transformation T from $\mathbf{R}^3$ to $\mathbf{R}^2$ with kernel W and image V, or explain why no such transformation exists.

- W is the z-axis and V = $\mathbf{R}^2$.
- W is the span of (1,1,1) and V is the line $x = y$.
- W is the span of the vectors (1,-1,1); (1,0,1), (0,1,0); and V is the line $x = -y$.