Spring 98 Exam 1 Solutions

1) The equations are $\left\{\begin{array}{l} w + n + s = 100 \\ .6w + .4n + .4s = w \\ .3w + .5n = n \\ .1w + .1n + .6s = s \end{array}\right\}$ and they can be rewritten $\left\{\begin{array}{l} w + n + s = 100 \\ -.4w + .4n + .4s = 0 \\ .3w - .5n = 0 \\ .1w + .1n - .4s = 0 \end{array}\right\}$

Solving by Gauss-Jordan gives W = 50, N = 30, and S = 20.

2)

- **A** is a rotation/dilation matrix, rotating counter-clockwise by 45 degrees and stretching by $\sqrt{2}$. So **A** turns the face into the second quadrant and stretches it a bit.
- $\mathbf{A}^4$ is also a rotation/dilation matrix, rotating counter-clockwise by 180 degrees and stretching by 4. So $\mathbf{A}^4$ flips the face over the x-axis and stretches it by 4.
- $\mathbf{A}^4$ is a diagonal matrix with -4's along the diagonal.
- $\mathbf{A}^{17}$ is rotation by 765 degrees and dilation by $\left(\sqrt{2}\right)^{17}$. So $\mathbf{A}^{17} = \begin{bmatrix} 256 & -256 \\ 256 & 256 \end{bmatrix}$.

3) $\mathbf{B} = \begin{bmatrix} -1/3 & 2/3 & -2/3 \\ 2/3 & -1/3 & -2/3 \\ -2/3 & -2/3 & -1/3 \end{bmatrix}$ $\quad \mathbf{C} = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$ $\quad \mathbf{E} = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$\mathbf{A} = \begin{bmatrix} 1/3 & 1/3 & -1/3 \\ 1/3 & 1/3 & -1/3 \\ -1/3 & -1/3 & 1/3 \end{bmatrix}$ $\quad \mathbf{D} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

4) The reduced row echelon form (rref) of **A** is $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. So a basis for the image is $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$.

A basis for the kernel is $\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$.

(5)

- $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ is one possible form.
- Not possible, as dim(image) + dim(kernel) must be 3, but dim(V) + dim(W) = 2.
- $\begin{bmatrix} 1 & 0 & -1 \\ -1 & 0 & 1 \end{bmatrix}$ is one possible form.