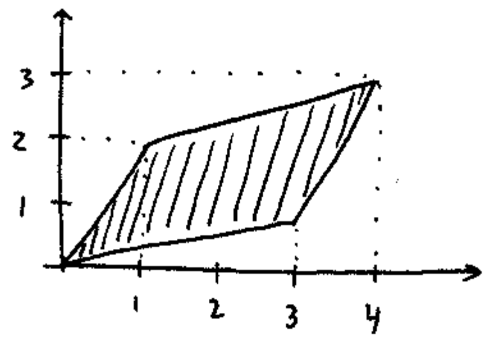
Math 21b Solutions HW #5

2.2

1) $T\begin{pmatrix}0\\0\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$, $T\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}3\\1\end{pmatrix}$, $T\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}4\\3\end{pmatrix}$, $T\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}1\\2\end{pmatrix}$



6) Let $\vec{u}$ be unit vector on L. $\vec{u}=\frac{1}{\sqrt{2^2+1^2+2^2}}\begin{pmatrix}2\\1\\2\end{pmatrix}=\frac{1}{3}\begin{pmatrix}2\\1\\2\end{pmatrix}$

Then, $\mathrm{Proj}_L\begin{pmatrix}1\\1\\1\end{pmatrix}=\left[\vec{u}\cdot\begin{pmatrix}1\\1\\1\end{pmatrix}\right]\vec{u}=\frac{5}{3}\cdot\vec{u}=\frac{5}{9}\begin{pmatrix}2\\1\\2\end{pmatrix}=\boxed{\begin{pmatrix}10/9\\5/9\\10/9\end{pmatrix}}$

7) By fact 2.2.7 $\mathrm{Ref}_L\begin{pmatrix}1\\1\\1\end{pmatrix}=2\left(\vec{u}\cdot\begin{pmatrix}1\\1\\1\end{pmatrix}\right)\vec{u}-\begin{pmatrix}1\\1\\1\end{pmatrix}$, w/ $\vec{u}$ same as in problem 6. $\therefore$ $\mathrm{Ref}_L\begin{pmatrix}1\\1\\1\end{pmatrix}=2\cdot\frac{5}{3}\cdot\frac{1}{3}\begin{pmatrix}2\\1\\2\end{pmatrix}-\begin{pmatrix}1\\1\\1\end{pmatrix}=\boxed{\begin{pmatrix}11/9\\1/9\\11/9\end{pmatrix}}$

10) Here, $\vec{u}=\frac{1}{\sqrt{4^2+3^2}}\begin{pmatrix}4\\3\end{pmatrix}=\begin{pmatrix}0.8\\0.6\end{pmatrix}$.

So, $T(\vec{x})=A\vec{x}=(\vec{u}\cdot\vec{x})\vec{u}=(0.8x_1+0.6x_2)\begin{pmatrix}0.8\\0.6\end{pmatrix}=\begin{pmatrix}0.64x_1+0.48x_2\\0.48x_1+0.36x_2\end{pmatrix}$

$=\begin{pmatrix}0.64&0.48\\0.48&0.36\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}$. $\therefore$ $\boxed{A=\begin{pmatrix}0.64&0.48\\0.48&0.36\end{pmatrix}}$

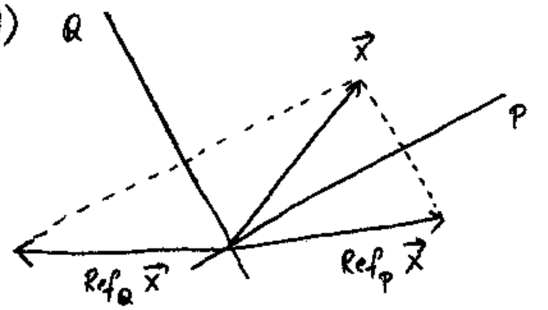
14) a) Generalizing the procedure of problem 10, we see that A is the matrix whose ijth entry $\boxed{A_{ij}=u_iu_j}$.

b) Sum of diagonal entries $\sum_{i=j}A_{ij}=\sum_{i=1}^{n}u_i^2=\boxed{1}$ as $\vec{u}$ is a unit vector.

2.2

20) $T(e_1) = e_1$, $T(e_2) = -e_2$, $T(e_3) = e_3$; $\therefore$ $\boxed{A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}$

41)



$\therefore$ $\boxed{Ref_Q \vec{x} = -Ref_P \vec{x}}$

47) Let $T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

a) $f(t) = \begin{pmatrix} a\cos t + b\sin t \\ c\cos t + d\sin t \end{pmatrix} \begin{pmatrix} -a\sin t + b\cos t \\ -c\sin t + d\cos t \end{pmatrix}$

$= (a\cos t + b\sin t)(-a\sin t + b\cos t) + (c\cos t + d\sin t)(-c\sin t + d\cos t)$

$\therefore$ f is cont, since cos, sin, and constant functions are cont., and sums and products of cont. functions are cont.

b) $f(\pi/2) = T\begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot T\begin{pmatrix} -1 \\ 0 \end{pmatrix} = -\left(T\begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot T\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right)$ as T is linear

$f(0) = T\begin{pmatrix} 1 \\ 0 \end{pmatrix} T\begin{pmatrix} 0 \\ 1 \end{pmatrix} = T\begin{pmatrix} 0 \\ 1 \end{pmatrix} T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = -f(\pi/2)$.

c) by b) either $f(\pi/2) = f(0) = 0$ OR $f(0)$ & $f(\pi/2)$ have different signs, & therefore zero lies b/w $f(0)$ & $f(\pi/2)$. As f is cont. by part a), intermediate value Thm. tells us there is a number c b/w 0 & $\pi/2$ s.t. $f(c) = 0$.

d) Note that $\begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$ and $\begin{bmatrix} -\sin t \\ \cos t \end{bmatrix}$ are perpendicular unit vectors for any t.

Let $\vec{v}_1 = \begin{pmatrix} \cos(c) \\ \sin(c) \end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix} -\sin(c) \\ \cos(c) \end{pmatrix}$, using number c from part c), then

$f(c) = T(\vec{v}_1) \cdot T(\vec{v}_2) = 0 \Rightarrow T(\vec{v}_1) \perp T(\vec{v}_2)$.