Math 21 b solutions HW #6

2.3

10) $$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 3 & 6 & 0 & 0 & 1 \end{array}\right)\begin{matrix} \\ -I \\ -I \end{matrix} \to \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 2 & 5 & -1 & 0 & 1 \end{array}\right)\begin{matrix} -II \\ \\ -2II \end{matrix} \to \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 2 & -1 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}\right)\begin{matrix} +III \\ -2III \\ \\ \end{matrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -3 & 1 \\ 0 & 1 & 0 & -3 & 5 & -2 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}\right) \quad \therefore \quad \boxed{\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{pmatrix}^{-1} = \begin{pmatrix} 3 & -3 & 1 \\ -3 & 5 & -2 \\ 1 & -2 & 1 \end{pmatrix}}$$

20) $$\left(\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 1 & 4 & 8 & 0 & 1 & 0 \\ 2 & 7 & 12 & 0 & 0 & 1 \end{array}\right)\begin{matrix} \\ -I \\ -2I \end{matrix} \to \left(\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & 1 & 5 & -1 & 1 & 0 \\ 0 & 1 & 6 & -2 & 0 & 1 \end{array}\right)\begin{matrix} -3II \\ \\ -II \end{matrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -12 & 4 & -3 & 0 \\ 0 & 1 & 5 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right)\begin{matrix} +12III \\ -5III \\ \\ \end{matrix} \to \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -8 & -15 & 12 \\ 0 & 1 & 0 & 4 & 6 & -5 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right)$$

$$\therefore \quad \boxed{\begin{aligned} x_1 &= -8y_1 - 15y_2 + 12y_3 \\ x_2 &= 4y_1 + 6y_2 - 5y_3 \\ x_3 &= -y_1 - y_2 + y_3 \end{aligned}}$$

26) Invertible. Consider $\boxed{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \sqrt[3]{y_2 - y_1} \\ y_1 \end{pmatrix}}$

30) $$\operatorname{Rref}\begin{pmatrix} 0 & 1 & b \\ -1 & 0 & c \\ -b & -c & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -c \\ 0 & 1 & b \\ 0 & 0 & 0 \end{pmatrix}$$

$\therefore$ The Matrix is not invertible regardless of the values of b & c.

40) If you apply an elementary row operation to a matrix w/ 2 equal columns, the resulting matrix will also have 2 equal columns.
$\therefore$ Rref(A) has 2 equal columns. $\therefore$ $\operatorname{Rref}(A) \neq I_n$.
$\therefore$ A is not invertible.

41) a) Invertible: the transformation is its own inverse.

b) Not invertible: It is neither true that the image of the transformation is all of $\mathbb{R}^3$ (it is in fact $\mathbb{R}^2$), nor that the transformation is one-to-one (2 different vectors can have the same projection onto a given plane).

c) Invertible: The inverse is a dilation by $1/5$.

d) Invertible: The inverse is a rotation about the same axis through the same angle in the opposite direction.

42) Permutation matrices are invertible as they can be row-reduced to I_n by simple row swaps. The inverse of a permutation matrix is also a permutation matrix since $[I_n \mid A^{-1}]$ can be obtained from $[A \mid I_n]$ by simple row swaps.