Janak Ramakrishnan

1. Since all of these are orthogonal to each other, we can take the dot product of each with itself and square root that, yielding $\sqrt{2+1+9+4} = 4$.

2. $\int_{-\pi}^{\pi} t dt = 0$ (all the coefficients don't change this). We know that $\frac{1}{\sqrt{2}}$ has length 1, so all we have to check is that $\sqrt{\frac{3}{2}}\frac{t}{\pi}$ does as well: $\frac{1}{\pi}\int_{-\pi}^{\pi} \frac{3}{2\pi^2}t^2 dt = \frac{3}{2\pi^3}\frac{t^3}{3}\Big]_{-\pi}^{\pi} = 1$. Yup. To find the projection of t^2, just find the components with respect to each of these functions. Since t^3 is odd, it has no component in the "direction" of t, but $\frac{1}{\pi}\int_{-\pi}^{\pi} \frac{t^2}{\sqrt{2}} dt = \frac{1}{3\sqrt{2}\pi}t^3\Big]_{-\pi}^{\pi} = \frac{\sqrt{2}\pi^2}{3}$

4. $\int_{-\pi}^{\pi} e^{at}\cos nt dt = \frac{-e^{at}\sin(nt)}{n}\Big]_{-\pi}^{\pi} + \frac{a}{n}\int_{-\pi}^{\pi} e^{at}\sin(nt)dt = \frac{a}{n}\left(\frac{e^{at}\cos(nt)}{n}\Big]_{-\pi}^{\pi} + -\frac{a}{n}\int_{-\pi}^{\pi} e^{at}\cos(nt)dt\right) = \left[\frac{a\cos nt}{n^2}e^{at}\right]_{-\pi}^{\pi} - \frac{a^2}{n^2}\int_{-\pi}^{\pi} e^{at}\cos nt dt$. Thus, $\int_{-\pi}^{\pi} e^{at}\cos(nt)dt = \frac{1}{n^2+a^2}a\cos(nt)e^{at}\Big]_{-\pi}^{\pi}$. This is

$$\frac{(-1)^n a\left(e^{a\pi} - e^{-a\pi}\right)}{n^2+a^2}$$

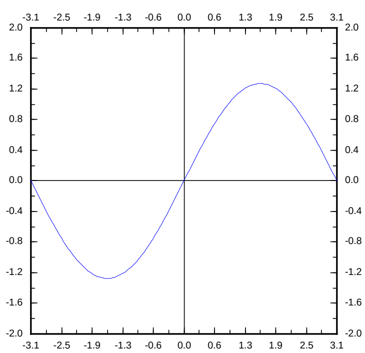
5. We know that cosh is even, so we only need to worry about cos and $\frac{1}{\sqrt{2}}$. $\left\langle \cosh(at), \frac{1}{\sqrt{2}}\right\rangle = \frac{e^{a\pi}-e^{-a\pi}}{\sqrt{2}a\pi}$. From 4, we know that $\langle \cosh(at), \cos(nt)\rangle = \frac{(-1)^n a\left(e^{a\pi}-e^{-a\pi}\right)}{\pi(n^2+a^2)}$ (notice that the answer in 4) is even w.r.t. a, and we have a $\frac{1}{2}$ floating around).

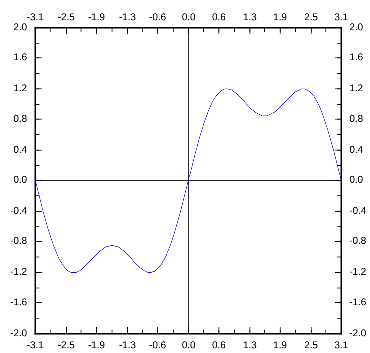
6. We would like to get a sum that looks like $\sum_{n=1}^{\infty}\frac{1}{n^2+a^2}$. Playing around with 5), we notice that if we evaluate $\cosh(a\pi)$ using Fact 1.2, we can cancel those pesky -1's, and we get $\cosh(a\pi) = \frac{e^{a\pi}-e^{-a\pi}}{2a\pi} + a\frac{\left(e^{a\pi}-e^{-a\pi}\right)}{\pi}\sum_{n=1}^{\infty}\frac{1}{n^2+a^2}$, so we get

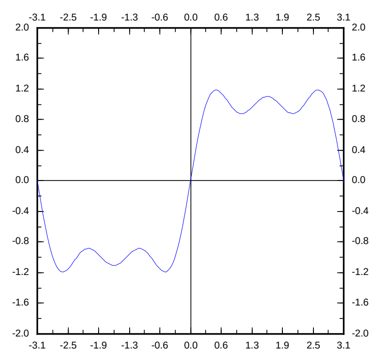
$$\sum_{n=1}^{\infty}\frac{1}{n^2+a^2} = \pi\frac{\left(e^{a\pi}+e^{-a\pi}\right) - \frac{e^{a\pi}-e^{-a\pi}}{\pi}}{2a\left(e^{a\pi}-e^{-a\pi}\right)} = \frac{1}{2a}\left(\pi\frac{e^{a\pi}+e^{a\pi}}{e^{a\pi}-e^{a\pi}} - 1\right)$$

26. f is odd, so we only have to do it with sin. We get $\frac{2}{\pi}\int_0^{\pi}\sin(nt)dt$

$$= \frac{-2}{\pi n}\cos(nt)\Big]_0^{\pi} = \begin{cases} 0 & \text{if n is even} \\ \frac{4}{\pi n} & \text{if n is odd} \end{cases}$$







28. From Fact 9.3.6, we get $\sqrt{\frac{1}{\pi}\left(\int_{-\pi}^{0}(-1)^2 dt + \int_0^{\pi} 1^2 dt\right)} = \sqrt{2} = \frac{16}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$, so

$$\frac{\sqrt{2}\pi^2}{4} = \sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}.$$