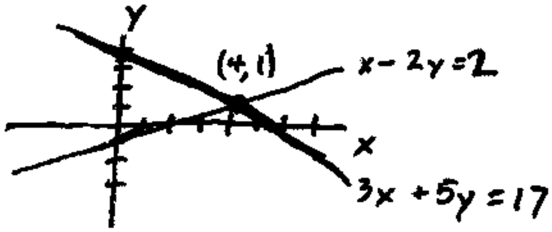
[redacted]

1.1 / 11, 16, 17, 20, 29, 36

11. $\left|\begin{array}{l} x - 2y = 2 \\ 3x + 5y = 17 \end{array}\right| \xrightarrow[R_2 - 3R_1]{R_1} \left|\begin{array}{l} x - 2y = 2 \\ 11y = 11 \end{array}\right| \xrightarrow[R_2/11]{R_1} \left|\begin{array}{l} x - 2y = 3 \\ y = 1 \end{array}\right| \xrightarrow[R_2]{R_1 + 2R_2} \left|\begin{array}{l} x = 4 \\ y = 1 \end{array}\right|$

$(x, y) = (4, 1)$



16. $\left|\begin{array}{l} x + 4y + z = 0 \\ 4x + 13y + 7z = 0 \\ 7x + 22y + 13z = 0 \end{array}\right| \xrightarrow[\text{III} - 7(\text{I})]{\text{II} - 4(\text{I})} \left|\begin{array}{l} x + 4y + z = 0 \\ -3y + 3z = 0 \\ -6y + 6z = 0 \end{array}\right| \xrightarrow[\text{III} - 2(\text{II})]{-3(\text{II})} \left|\begin{array}{l} x + 4y + z = 0 \\ y - z = 0 \end{array}\right|$

$\xrightarrow{\text{I} - 4(\text{II})} \left|\begin{array}{l} x + 5z = 0 \\ y - z = 0 \end{array}\right|$ let $z = t \Rightarrow \left.\begin{array}{l} x = -5t \\ y = t \\ z = t \end{array}\right\} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = t\begin{pmatrix} -5 \\ 1 \\ 1 \end{pmatrix}$

→ solution is a line

17. $\left|\begin{array}{l} x + 2y = a \\ 3x + 5y = b \end{array}\right| \rightarrow \left|\begin{array}{l} x + 2y = a \\ -y = b - 3a \end{array}\right| \rightarrow \left|\begin{array}{l} x = 2b - 5a \\ y = 3a - b \end{array}\right|$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -5a + 2b \\ 3a - b \end{pmatrix}$$

20. $\left|\begin{array}{l} a = 1000 + .1b \\ b = 780 + .2a \end{array}\right| \rightarrow \left|\begin{array}{l} a - .1b = 1000 \\ -.2a + b = 780 \end{array}\right| \rightarrow \left|\begin{array}{l} a - .1b = 1000 \\ .98b = 980 \end{array}\right|$

$\left|\begin{array}{l} a - .1b = 1000 \\ b = 1000 \end{array}\right| \rightarrow \left|\begin{array}{l} a = 1100 \\ b = 1000 \end{array}\right|$

$$\boxed{\begin{array}{l} a = 1100 \\ b = 1000 \end{array}}$$

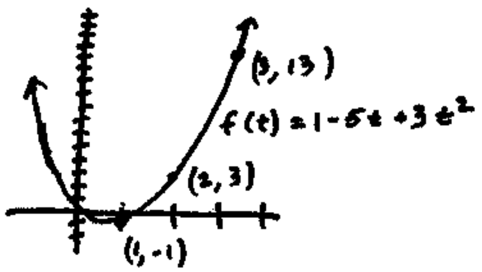
1.1 / 11, 16, 17, 20, 29, 36 (page 2)

29. $f(t) = a + bt + ct^2$

[redacted]

$$\left|\begin{array}{l} a + b(1) + c(1)^2 = -1 \\ a + b(2) + c(2)^2 = 3 \\ a + b(3) + c(3)^2 = 13 \end{array}\right| \rightarrow \left|\begin{array}{l} a + b + c = -1 \\ a + 2b + 4c = 3 \\ a + 3b + 9c = 13 \end{array}\right| \rightarrow \left|\begin{array}{r} a + b + c = -1 \\ b + 3c = 4 \\ 2b + 8c = 14 \end{array}\right|$$

$$\left|\begin{array}{r} a \quad -2c = -5 \\ b + 3c = 4 \\ 2c = 6 \end{array}\right| \rightarrow \left|\begin{array}{r} a \quad = 1 \\ b \quad = -5 \\ c = 3 \end{array}\right| \Rightarrow \boxed{f(t) = 1 - 5t + 3t^2}$$



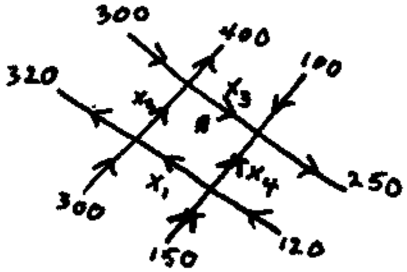
36. Let b = Boris' money, m = Marina's money, c = cost of choc. bar

$$\left|\begin{array}{l} b + \tfrac{1}{2} m = c \\ \tfrac{1}{2} b + m = 2c \end{array}\right| \rightarrow \left|\begin{array}{r} b + \tfrac{1}{2} m = c \\ \tfrac{3}{4} m = \tfrac{3}{2} c \end{array}\right| \rightarrow \left|\begin{array}{r} b \quad = 0 \\ m = 2c \end{array}\right|$$

$\boxed{\text{Boris has no money}}$

1.2 / 6, 9, 20, 33, 37, 42 (page 3)

42. Let x_1, x_2, x_3, x_4 be the traffic volume at the four locations,



$$\left\{\begin{array}{l} x_1 + 300 = 320 + x_2 \\ x_2 + 300 = 400 + x_3 \\ x_3 + x_4 + 100 = 250 \\ 150 + 120 = x_1 + x_4 \end{array}\right\} \rightarrow \left\{\begin{array}{lllll} x_1 & -x_2 & & & = 20 \\ & x_2 & -x_3 & & = 100 \\ & & x_3 & +x_4 & = 150 \\ & & & x_4 & = 270 \end{array}\right\}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 20 \\ 0 & 1 & -1 & 0 & 100 \\ 0 & 0 & 1 & 1 & 150 \\ 1 & 0 & 0 & 1 & 270 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 270 \\ 0 & 1 & 0 & 1 & 250 \\ 0 & 0 & 1 & 1 & 150 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{pmatrix} 270 \\ 250 \\ 150 \\ 0 \end{pmatrix} + t \begin{pmatrix} -1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$$

one possible scenario: $x_1 = 270$, $x_2 = 250$, $x_3 = 150$, $x_4 = 0$

Ranges: all values must always be positive or zero

So... $0 \le x_4 \le 150$

All other values are highest when x_4 is zero and lowest when it is 150

So...

$$120 \le x_1 \le 270$$
$$100 \le x_2 \le 250$$
$$0 \le x_3 \le 150$$

1.2 / 6, 9, 20, 33, 37, 42

6.

$$\begin{array}{ccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & -7 & 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 0 & -2 & 2 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{array}\right]$$ system is already in rref

$$\left|\begin{array}{l} x_1 = 3 + 7x_2 - x_5 \\ x_3 = 2 + 2x_5 \\ x_4 = 1 - x_5 \end{array}\right| \quad \text{let } \left.\begin{array}{l} x_2 = s \\ x_5 = t \end{array}\right\} \Rightarrow \boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{pmatrix} 3 \\ 0 \\ 2 \\ 1 \\ 0 \end{pmatrix} + s\begin{pmatrix} 7 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} -1 \\ 0 \\ 2 \\ -1 \\ 1 \end{pmatrix}}$$

9.

$$\left[\begin{array}{cccccc|c} 0 & 0 & 0 & 1 & 2 & -1 & 2 \\ 1 & 2 & 0 & 0 & 1 & -1 & 0 \\ 1 & 2 & 2 & 0 & -1 & 1 & 2 \end{array}\right] \to \left[\begin{array}{cccccc|c} 1 & 2 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 2 & -1 & 2 \\ 1 & 2 & 2 & 0 & -1 & 1 & 2 \end{array}\right] \to \left[\begin{array}{cccccc|c} 1 & 2 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 2 & -1 & 2 \\ 0 & 0 & 2 & 0 & -2 & 2 & 2 \end{array}\right]$$

$$\left[\begin{array}{cccccc|c} 1 & 2 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 2 & 0 & -2 & 2 & 2 \\ 0 & 0 & 0 & 1 & 2 & -1 & 2 \end{array}\right] \to \left[\begin{array}{cccccc|c} 1 & 2 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 & -1 & 2 \end{array}\right] \Rightarrow \left\{\begin{array}{l} x_1 = -2x_2 - x_5 + x_6 \\ x_3 = 1 + x_5 - x_6 \\ x_4 = 2 - 2x_5 + x_6 \end{array}\right\}$$

let $x_2 = r$, $x_5 = s$, $x_6 = t$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 2 \\ 0 \\ 0 \end{pmatrix} + r\begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + s\begin{pmatrix} -1 \\ 0 \\ 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

20. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & k \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $k \in \mathbb{R}$

$\boxed{\text{four}}$

1.2 / 6, 9, 20, 33, 37, 42 (page 2)

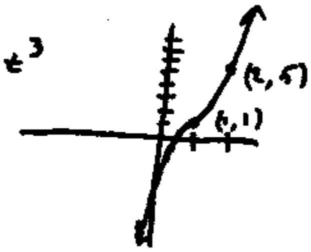
33. $f(t) = a + bt + ct^2 + dt^3$

$f'(t) = b + 2ct + 3dt^2$

$$\begin{aligned} a + b + c + d &= 1 \\ a + 2b + 4c + 8d &= 5 \\ b + 2c + 3d &= 2 \\ b + 4c + 12d &= 9 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 & 5 \\ 0 & 1 & 2 & 3 & 2 \\ 0 & 1 & 4 & 12 & 9 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 7 & 4 \\ 0 & 1 & 2 & 3 & 2 \\ 0 & 1 & 4 & 12 & 9 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 0 & -2 & -6 & -3 \\ 0 & 1 & 3 & 7 & 4 \\ 0 & 0 & -1 & -4 & -2 \\ 0 & 0 & 1 & 5 & 5 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 1 \\ 0 & 1 & 0 & -5 & -2 \\ 0 & 0 & 1 & 4 & 2 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

$$\to \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -5 \\ 0 & 1 & 0 & 0 & 13 \\ 0 & 0 & 1 & 0 & -10 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \Rightarrow f(t) = -5 + 13t - 10t^2 + 3t^3$$



37. $$\begin{cases} x_1 = .2x_2 + .3x_3 + 320 \\ x_2 = .1x_1 + .4x_3 + 90 \\ x_3 = .2x_1 + .5x_2 + 150 \end{cases} \to \begin{aligned} x_1 - .2x_2 - .3x_3 &= 320 \\ -.1x_1 + x_2 - .4x_3 &= 90 \\ -.2x_1 - .5x_2 + x_3 &= 150 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & -.2 & -.3 & 320 \\ -.1 & 1 & -.4 & 90 \\ -.2 & -.5 & 1 & 150 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cccc} 1 & 0 & 0 & 500 \\ 0 & 1 & 0 & 300 \\ 0 & 0 & 1 & 400 \end{array}\right] \quad \boxed{\begin{aligned} x_1 &= 500 \\ x_2 &= 300 \\ x_3 &= 400 \end{aligned}}$$

1.3 / 1, 14, 25, 34, 48, 50

1. a. last row indicates $0 = 1 \Rightarrow$ inconsistent system
$\boxed{\text{no solutions}}$

b. unique solution

$$x = 5$$
$$y = 6$$

c. Infinite. solution is linear. First variable can be chosen freely.

14. $$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} = -1\begin{bmatrix} 1 \\ 2 \end{bmatrix} + 2\begin{bmatrix} 2 \\ 3 \end{bmatrix} + 1\begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1\cdot(-1) + 2\cdot 2 + 3\cdot 1 \\ 2\cdot(-1) + 3\cdot 2 + 4\cdot 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$$

25. $A\vec{x} = \vec{b}$ is inconsistent

$\Rightarrow \operatorname{rank}([A \vdots \vec{b}\,]) > \operatorname{rank}(A)$

$\operatorname{rank}([A \vdots \vec{b}\,]) \leq 4$ (matrix has only 4 rows)

$\Rightarrow \operatorname{rank}(A) < 4$

$\Rightarrow$ rref(A) has at least one row of zeroes

$\Rightarrow A\vec{x} = \vec{c}$ could have infinitely many solutions (as in $\vec{c} = \vec{0}$) or no solutions (as in $\vec{c} = \vec{b}$) but can not have a unique solution.

1.3/ 1, 14, 25,
34, 48, 50 (page 2)

34. a. $A\vec{e}_1 = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 1\cdot\begin{bmatrix} a \\ d \\ g \end{bmatrix} + 0\cdot\begin{bmatrix} b \\ e \\ h \end{bmatrix} + 0\cdot\begin{bmatrix} c \\ f \\ k \end{bmatrix} = \begin{bmatrix} a \\ d \\ g \end{bmatrix}$

by same reasoning

$A\vec{e}_2 = \begin{bmatrix} b \\ e \\ h \end{bmatrix}$

$A\vec{e}_3 = \begin{bmatrix} c \\ f \\ k \end{bmatrix}$

b. $B = \begin{bmatrix} \uparrow & \uparrow & \uparrow \\ \vec{v}_1 & \vec{v}_2 & \vec{v}_3 \\ \downarrow & \downarrow & \downarrow \end{bmatrix}$

$B\vec{e}_1 = 1\cdot\vec{v}_1 + 0\cdot\vec{v}_2 + 0\cdot\vec{v}_3 = \vec{v}_1$
$B\vec{e}_2 = \vec{v}_2$
$B\vec{e}_3 = \vec{v}_3$

48. a. we know that $A\vec{x}_1 = \vec{b}$ and $A\vec{x}_h = \vec{0}$

$\Rightarrow \quad A\vec{x}_1 + A\vec{x}_h = \vec{b} + \vec{0}$ (add eq'ns)

$A(\vec{x}_1 + \vec{x}_h) = \vec{b}$

$\Rightarrow \quad \vec{x}_1 + \vec{x}_h$ is a solution to $A\vec{x} = \vec{b}$

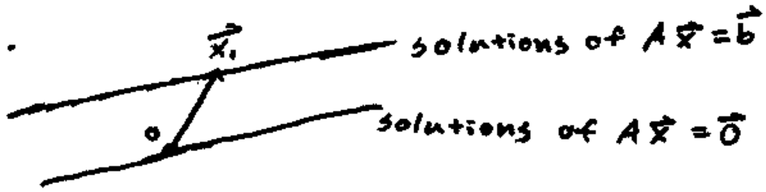
b. we know that $A\vec{x}_1 = \vec{b}$ and $A\vec{x}_2 = \vec{b}$

$\Rightarrow \quad A\vec{x}_2 - A\vec{x}_1 = \vec{b} - \vec{b}$ (subtract eq'ns)

$A(\vec{x}_2 - \vec{x}_1) = \vec{0}$

$\Rightarrow \quad \vec{x}_2 - \vec{x}_1$ is a solution to $A\vec{x} = \vec{0}$

c.



→ adding any solution of $A\vec{x} = \vec{0}$ to $\vec{x}_1$ gives us a solution to $A\vec{x} = \vec{b}$, so the solution is paralel to the $A\vec{x} = \vec{0}$ solutions, running through $\vec{x}_1$

1.3 / 1, 14, 25, 34, 48, 50 (page 3)

50. A is a 4×3 matrix

$\Rightarrow \operatorname{rank} A \le 3$

$\operatorname{rank} [A \vdots \vec{b}] = 4$

$\Rightarrow \operatorname{rank} [A \vdots \vec{b}] > \operatorname{rank} A$

$\Rightarrow A\vec{x} = \vec{b}$ is ~~an~~ inconsistent

$\boxed{\text{no solutions}}$