Review for Midterm 1

Consider a system of linear equations:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots &+ a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots &+ a_{2n}x_n = b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots &+ a_{mn}x_n = b_m \end{aligned}$$

We have roughly 3 ways of approaching this system.

(A) Gaussian elimination

Look at the augmented matrix

$$\left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ & \vdots & & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{array}\right]$$

and place it row-reduced echelon form (RREF)

By looking at RREF, we can see if a system is consistent or inconsistent.

A leading 1 in the augmented column means no solutions

$$\left[\begin{array}{ccccc|c} 0 & 0 & 0 & \cdots & 0 & 1 \end{array}\right]$$

Otherwise...

If every variable is a leading variable (that is, if every variable column has a leading 1), we have a unique solution

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & * \\ 0 & 1 & 0 & 0 & * \\ 0 & 0 & 1 & 0 & * \\ 0 & 0 & 0 & 1 & * \end{array}\right]$$

If there are columns without leading 1s, we can make the variables corresponding to these columns anything we want, and then solve for the other variables; so we have infinitely many solutions.

$$\left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 5 \\ 0 & 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \qquad \begin{aligned} x_4 &= 3 \\ x_3 &= a \quad (\text{parameter}) \\ x_2 &= 2-a \\ x_1 &= 5-2a \end{aligned}$$

The number of leading 1's in a RREF'd matrix is the rank of the matrix.

B Linear transformation

We could also write the system as

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ & \vdots & & \\ a_{m1} & \cdots & & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ \vdots \\ b_m \end{bmatrix}$$

or, more concisely, as $A\vec{x} = \vec{b}$. A is a matrix of size $m \times n$; $\vec{x}$ is a vector in $\mathbb{R}^n$; $\vec{b}$ is a vector in $\mathbb{R}^m$.

A can be considered a transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$. You feed A a vector in $\mathbb{R}^n$, you multiply A by that vector, you get a vector in $\mathbb{R}^m$. You've transformed the first vector into the second.

Matrix transformations ~~are~~ are linear transformations

$$① \quad A(\vec{x}+\vec{y}) = A\vec{x} + A\vec{y}$$

$$② \quad A(c\vec{x}) = cA\vec{x} \qquad \text{(c is a scalar, i.e., real number)}$$

Also, every linear transformation can be written as a matrix. So any function from $\mathbb{R}^n$ to $\mathbb{R}^m$ with properties ① and ② has a matrix form.

Some examples of linear transformations are:

- identity (sends every vector to itself)
- zero (sends every vector to $\vec{0}$)
- rotations
- reflections over a line in $\mathbb{R}^2$
- reflections over a plane in $\mathbb{R}^3$
- orthogonal projection to a line in $\mathbb{R}^2$
- orthogonal projection to a plane in $\mathbb{R}^3$
- shears in $\mathbb{R}^2$

Given a transformation T, you may want to (or have to) write down it's matrix. To do this, figure out what the transformation T does to the standard basis vectors. In $\mathbb{R}^2$, the standard basis vectors are $\vec{e}_1 = \begin{bmatrix}1\\0\end{bmatrix}$, $\vec{e}_2 = \begin{bmatrix}0\\1\end{bmatrix}$. In $\mathbb{R}^3$, they are

$\vec{e}_1 = \begin{bmatrix}1\\0\\0\end{bmatrix}$, $\vec{e}_2 = \begin{bmatrix}0\\1\\0\end{bmatrix}$, $\vec{e}_3 = \begin{bmatrix}0\\0\\1\end{bmatrix}$. The columns of the matrix are $T\vec{e}_1$, $T\vec{e}_2$, etc.

So for example, let's say you wanted to find the matrix for a reflection over the x-axis in $\mathbb{R}^2$. $\vec{e}_1$ stays the same, $\vec{e}_2$ becomes $\begin{bmatrix}0\\-1\end{bmatrix}$.

So the matrix is $\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$.

Well, what does this have to do with solving systems of linear equations? A little terminology is helpful:

Image The image of a linear transformation T is the set of all vectors $T\vec{x}$. Basically, it's the "range" of the function, the set of all possible outputs.

Kernel The kernel of a linear transformation T is the set of all vectors $\vec{x}$ s.t. $T\vec{x} = \vec{0}$. Basically, it's everything that goes to $\vec{0}$.

Now if we have an equation

$$A\vec{x} = \vec{b}$$

we know:

1) if $\vec{b}$ is not in the image of A, there are no solutions
2) there is a unique solution only if the kernel of A consists only of $\vec{0}$

1) follows from the definition of image.
2) is a bit trickier. Say we have a vector $\vec{x}_1$ s.t. $A\vec{x}_1 = \vec{b}$. And say we have a vector $\vec{x}_0$ s.t. $A\vec{x}_0 = \vec{0}$, $\vec{x}_0 \neq \vec{0}$. Then $A(\vec{x}_1 + c\vec{x}_0) = A\vec{x}_1 + A(c\vec{x}_0) = A\vec{x}_1 + cA\vec{x}_0 = A\vec{x}_1 = \vec{b}$, so we have infinitely many solutions to $A\vec{x} = \vec{b}$, if we have one at all.

If A is a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$, finding solutions to $A\vec{x} = \vec{b}$ is similar to finding a "reverse transformation" from $\mathbb{R}^m$ to $\mathbb{R}^n$ that gives you $\vec{x}$ when you plug in $\vec{b}$.

~~It turns out that such inverses exist only when~~

~~If we can find a good inverse~~

Say we wanted to find an inverse for $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

Here's how we could do it: row-reduce

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right] \qquad \begin{aligned} x_1 + 2x_2 &= y_1 \\ 3x_1 + 4x_2 &= \phantom{y_1 + {}} y_2 \end{aligned}$$

to

$$\left[\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & 1 & 3/2 & -1/2 \end{array}\right] \qquad \begin{aligned} x_1 \phantom{{}+x_2} &= -2y_1 + y_2 \\ x_2 &= 3/2\, y_2 + -1/2\, y_2 \end{aligned}$$

The matrix $\begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$ is the inverse.

If we couldn't get the identity matrix on the left side of rref, then we wouldn't be able to get an inverse, since we wouldn't be able to uniquely solve for x_1, x_2 in terms of y_1, y_2. Since the left side is simply the rref of the original matrix, A is invertible exactly when $\text{rref}(A) = I_2$.

The above procedure generalizes for matrices of all sizes.

This means that only square matrices can be invertible, since the rref must be square.

We can also characterize invertible matrices by their kernels and images: an $n \times n$ matrix is invertible if and only if the image is $\mathbb{R}^n$ and the kernel is $\{\vec{0}\}$. (We will soon see that these conditions are the same.)

(c) Linear combination of vectors

We can write our original system (see page 1) in this form:

$$x_1 \begin{bmatrix} a_{11} \\ \vdots \\ a_{m1} \end{bmatrix} + x_2 \begin{bmatrix} a_{12} \\ \vdots \\ a_{m2} \end{bmatrix} + \dots + x_n \begin{bmatrix} a_{1n} \\ \vdots \\ a_{mn} \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix}$$

Now some terminology:

A vector space is a set of vectors such that

1) $\vec{0}$ is in the space
2) if $\vec{v}$ is in the space, $k\vec{v}$ is in the space (k is any scalar)
3) if $\vec{v}$ and $\vec{w}$ are in the space, $\vec{v}+\vec{w}$ is in the space

Vector spaces are nice and closed: you can't fall out of a space by adding up stuff that's in the space.
Our canonical examples of vector spaces are $\mathbb{R}^2, \mathbb{R}^3, \mathbb{R}^4, \dots$; that is, sets of n-vectors.

A subspace V of a vector space W is
a) a subset of W
b) a vector space in its own right

So subspaces are vector spaces inside other vector spaces.
To show that something is a vector space, simply show that the properties in the definition hold; same goes for showing that something is a subspace. It's a step-by-step process. To show that something is not a vector space, come up with vectors in the space that don't observe the definition.

One way to come up with a vector space is to take a bunch of vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ in $\mathbb{R}^n$, and look at vectors in the form

$$c_1\vec{v}_1 + c_2\vec{v}_2 + \ldots + c_n\vec{v}_n \; ; \; c_1, \ldots, c_n \text{ scalars}$$

These are linear combinations of $\vec{v}_1, \ldots, \vec{v}_n$.
The set of these vectors is a subspace of $\mathbb{R}^n$.

1) It's a subset: if $\vec{v}_1, \ldots, \vec{v}_n$ are in $\mathbb{R}^n$, $c_1\vec{v}_1, \ldots, c_n\vec{v}_n$ are all in $\mathbb{R}^n$ (because $\mathbb{R}^n$ is a vector space) and so their sum is in $\mathbb{R}^n$ (also because $\mathbb{R}^n$ is a vector space)

2) $\vec{0} = 0\vec{v}_1 + 0\vec{v}_2 + \ldots + 0\vec{v}_n$ is in the space

3) if $\vec{v}$ is in the space, $\vec{v} = c_1\vec{v}_1 + \ldots + c_n\vec{v}_n$ (every vector in the space has this form), so $k\vec{v} = kc_1\vec{v}_1 + kc_2\vec{v}_2 + \ldots + kc_n\vec{v}_n$ is also in the space

4) if $\vec{v} = c_1\vec{v}_1 + c_2\vec{v}_2 + \ldots + c_n\vec{v}_n$ and $\vec{w} = d_1\vec{v}_1 + \ldots + d_n\vec{v}_n$, then $\vec{v} + \vec{w} = (c_1+d_1)\vec{v}_1 + (c_2+d_2)\vec{v}_2 + \ldots + (c_n+d_n)\vec{v}_n$ is in the space

(This is the general process you would follow if asked to show that something was a subspace.)

The vector space we come up with is called $\text{Span}(\vec{v}_1, \ldots, \vec{v}_n)$; it is the space spanned by the vectors $\vec{v}_1, \ldots, \vec{v}_n$.

So we can see now that our original system has solutions exactly when $\begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix}$ is a linear combination of $\begin{bmatrix} a_{11} \\ \vdots \\ a_{m1} \end{bmatrix}, \ldots, \begin{bmatrix} a_{1n} \\ \vdots \\ a_{mn} \end{bmatrix}$
is in the span of

or, in other words, when $\vec{b}$ is a linear combination of the column vectors of A (going back to the matrix notation in section (B))

We can also say, equivalently, that the image of A is the span of its columns.

Some more terminology ...

~~If~~ We know that the vectors $\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}$ span $\mathbb{R}^2$ – every vector in $\mathbb{R}^2$ can be written as a linear combination of these vectors — but somehow these vectors are redundant. $\begin{bmatrix}1\\0\end{bmatrix}$ and $\begin{bmatrix}0\\1\end{bmatrix}$ would work just fine.

A linearly independent set is not redundant, while a linearly dependent set is. We ~~to~~ call $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ linearly independent if $c_1\vec{v}_1 + c_2\vec{v}_2 + \ldots + c_n\vec{v}_n = 0$ only when $c_1 = c_2 = \cdots = c_n = 0$. They are linearly dependent if there is a nontrivial set of scalars that work. If the vectors are linearly dependent, then one can be written as a linear combination of the others (hence the redunancy).

Putting some ideas together:

- A matrix has kernel exactly equal to $\{\vec{0}\}$ if and only if its columns are linearly independent

If a set of vectors spans a vector space and is also linearly independent, it forms a <u>basis</u> for the vector space. The number of vectors in a ~~subs~~ basis is the <u>dimension</u> of the subspace.

It should also be observed that the image and the kernel of a linear transformation are both vector spaces.

To find a basis for the kernel, we use Gaussian elimination. For example, if our matrix is $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 4 & 1 & 6 \\ 3 & 6 & 0 & 3 \\ 3 & 6 & 1 & 7 \end{bmatrix}$

we row-reduce $\left[\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 0 \\ 2 & 4 & 1 & 6 & 0 \\ 3 & 6 & 0 & 3 & 0 \\ 3 & 6 & 1 & 7 & 0 \end{array}\right]$ to $\left[\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

We use parameters for the non-leading variables: $x_2 = a$, $x_4 = b$

We write the leading variables in terms of the parameters: $x_1 = -2a - b$, $x_3 = -4b$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2a - b \\ a \\ -4b \\ b \end{bmatrix} = a \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + b \begin{bmatrix} -1 \\ 0 \\ -4 \\ 1 \end{bmatrix}$$

So the kernel has basis $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -4 \\ 1 \end{bmatrix} \right\}$. Its dimension is the number of non-leading variables in RREF of the matrix.

To find a basis for the image, we use the fact that the image is spanned by the columns of the matrix. So the key is finding a linearly independent subset of columns that still span. To do this, find the RREF. The linearly independent ~~rows~~ columns correspond to the leading variables. So in the example above, we know that $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$ is a basis for the image. Since the dimension of the image is the number of leading variables, ~~we know~~ and the dimension of the kernel is the number of non-leading variables, we know

$$\begin{aligned} \dim(\text{Image } A) + \dim(\text{Kernel } A) &= \#\text{ columns of } A \\ &= n, \text{ if } A \text{ maps from } \mathbb{R}^n \to \mathbb{R}^m. \end{aligned}$$

Bridget Tenner

A bit of review for Math 21b Midterm 1 on Monday, October 25, 1999

(This review covers through section 3.2, but does not include the final section or two that may be on the midterm)

Things to keep in mind

Row – the set of elements comprising a horizontal line in a matrix

Column – the set of elements comprising a vertical line in a matrix

Entry – a number or variable occupying a single position in a matrix

Row vector – the ordered set of elements in a specific row

Vector – the ordered set of elements in a specific column

Number/scalar – interchangeable terms

Coefficient matrix – the matrix whose entries are the coefficients of a system of linear equations, where each row represents a different equation and each column represents the coefficients of a different variable

Augmented matrix – adding a column vector to the right side of a coefficient matrix where that column vector represents the vector b in the equation Ax = b

Gaussian elimination – a method of reducing any matrix to a form readily displaying much information about the transformation represented in the original matrix

rref – "reduced row echelon form"; a matrix where each row's first nonzero entry is a 1 (called a "leading 1"), and where each column containing a leading 1 contains 0's everywhere else; the form of a matrix that Gaussian elimination leads to

Consistent/inconsistent – the situation of either having a solution to an equation or not having any (respectively); there can be either 0, 1, or infinitely many solutions to an equation

Leading "1" (as opposed to a leading variable) – when dealing with a coefficient matrix, all the leading 1's are leading variables, but when dealing with an augmented matrix a leading 1 in the last column does not represent any variable, but indicates an inconsistent system; if all the variables are leading 1's then there is a unique solution to the equation, but if any are nonleading then there are infinitely many solutsion

Rank – the number of leading 1's in an rref'd matrix, not necessarily the number of leading variables

Linear combination – something is called a linear combination of $v_1, v_2, \ldots, v_n$ if it can be written as $k_1v_1 + k_2v_2 + \ldots + k_nv_n$ where $k_1, k_2, \ldots, k_n$ are constants

Vector/matrix – a matrix is a rectangular array of numbers; a matrix is often described in terms of its dimension as mxn where m is the number of rows (number of entries in a column) and n is the number of columns (number of entries in a row); a vector is simply an mx1 matrix

$\mathbf{R}^n$ – is the space defined by ordered "n-tuples" of real nubers (e.g. $(x_1, x_2, \ldots, x_n)$)

Linear transformation – a function T from $\mathbf{R}_n$ to $\mathbf{R}_m$ where there is an mxn matrix A such that T(x) = A(x); in other words, a function whose operation on x can be written as some matrix A times x

Inverse linear transformation (when it exists) – a transformation that "undoes" what ever the transformation T does to a vector, a way of getting back to the original vector; a matrix A (and hence a transformation T) has an inverse if and only if A is an nxn matrix (it's square) and rank(A) = n

Identity matrix I_n – the nxn matrix consisting of all 0's except the main diagonal which is all 1's

Zero matrix – any matrix consisting entirely of 0's

Geometric representation – a way of interpreting geometrically what a matrix does to a vector; in other words, think back to the idea of transformation and ask "what happens to a vector x after I transform it?"

Standard (unit) vectors (e_1, e_2, ...) – these are vectors that represent the direction of each coordinate axis; in other words, in 3-dimensional space, $e_1 = (1, 0, 0)$, $e_2 = (0, 1, 0)$, and $e_3 = (0, 0, 1)$; note that $a_1e_1 + a_2e_2 + \ldots + a_ne_n$ is the matrix $<a_1\ a_2 \ldots a_n>$ and the column vectors of a matrix A are equal to $A(e_1)$, $A(e_2)$, etc.

Linear characterization of linear transformation – determining exactly what sort of geometric interpretation to associate with a given transformation; the possibilities are:

Dilation – changing the magnitude of a vector but maintaining its direction

Rotation – changing the direction of a vector but maintaining its direction

Shear – a transformation that moves each point in the direction of some line through the origin

Orthogonal projection – drawing a perpendicular line from the vector to some specific line and making the image of the original vector wherever that perpendicular intersects the line

How to decide when a linear transformation is invertible – when each output of the transformation came from a unique input (rank = n)

How to find an inverse – augment the matrix A with the identity matrix of the same size and row reduce A, making sure to apply the same operations to the identity matrix as to A; when A is reduced to the identity (which it must be in order for A to have an inverse, this is equivalent to saying rank(A) = n), the matrix on the right hand side, where the identity was originally, is the inverse of A, denoted A^{-1}

Composition of functions – the process of first applying one function (or transformation) and then applying a second function (or transformation) to the outcome of the first; when writing compositions work backwards from right to left ($f \circ g (x)$ means first do g, then f because this can also be written as $f (g (x))$

How to compute a matrix product – first be aware that not all matrices can be multiplied, they need to be of compatible sizes, namely an mxn matrix can only be multiplied on the right by an nxp matrix, and the outcome will be an mxp matrix; to determine the entry in the ith row and jth column of the product AB, consider the ith row of A and the jth column of B and add the products of their corresponding terms (multiply the first terms in each, add this to the product of the second terms in each, add this to the product of the third terms in each, etc.)

Properties of matrix multiplication – assosicativity A(BC) = (AB)C, but not always commutativity (AB = BA)

$(AB)^{-1} = B^{-1}A^{-1}$

Span – the span of a set of vectors is the set of all linear combinations of those vectors

Image of a linear transformation – this is equivalent to the span of the column vectors of the matrix A for the transformation T, and occasionally called the column space of A; it is all possible outputs of the transformation; Im(A) has the following properties:

1. The zero vector is always in the image because A(0) = 0
2. If v_1 and v_2 are in Im(A) then $(v_1 + v_2)$ is in Im(A)
3. If v is in Im(A) then kv is in Im(A), where k is any scalar

Kernel of a linear transformation – also called the nullspace; this is the set of all vectors which get mapped by T (or equivalently A) to the zero vector; ker(A) has the following properties:

1. The zero vector is always in the kernel because A(0) = 0

2. If v_1 and v_2 are in Ker(A) then $(v_1 + v_2)$ is in Ker(A)
3. If v is in Ker(A) then kv is in Ker(A), where k is any scalar

The invertible case – if f, a function from X to Y, is invertible, then Im(f) = Y; an nxn matrix A is invertible if and only if Ax = b has a unique solution for all vectors b, rref(A) = I, rank(A) = n, Im(A) = $\mathbf{R}^n$ and Ker(A) = {0} (does not contain anything other than the zero vector), the column vectors of A are linearly independent

Space – a mathematical term that indicates structure and dimension

Subspace – a subset W of a space such that 0 is in W, if w_1 and w_2 are in W then $w_1 + w_2$ is in W, and if w is in W then kw is in W for any scalar k; note: {0} and $\mathbf{R}^n$ are always subspaces of $\mathbf{R}^n$

Linear dependence – a set of vectors $v_1, \ldots, v_n$ is said to be linearly dependent if one of the vectors can be written as a linear combination of the rest of the vectors; equivalently, a set of vectors is linearly dependent if some nontrivial linear combination of them (nontrivial meaning that not all the coefficients are 0) equals the zero vector; note that if v is a linear combination of $v_1; \ldots, v_n$ then Span($v, v_1, \ldots, v_n$) = Span ($v_1, \ldots, v_n$)

Linear independence – a set of vectors $v_1, \ldots, v_n$ is said to be linearly independent if none of the vectors can be written as a linear combination of any of the others

Basis – a set of vectors that are linearly independent and span the space

Practice Problems

(You certainly do not have to do all of these in order to be prepared for the midterm, but it would not be a bad idea to look through them. Also, all the problems are odd-numbered problems, so their answers should be in the back of the book)

Section 1.1: 15, 17, 25, 31, 37
Section 1.2: 9, 29, 31, 39
Section 1.3: 7, 17, 19, 23, 27, 29, 43, 51
Section 2.1: 7, 11, 13, 33, 45
Section 2.2: 13, 17, 23, 27, 33, 43, 51
Section 2.3: 5, 19, 29, 35, 47
Section 2.4: 7, 17, 23, 35, 47, 67, 71, 81
Section 3.1: 7, 23, 37, 43, 51, 53
Section 3.2: 1, 5, 17, 23, 27, 37, 49

More Questions/Problems

1. T is a linear transformation from V to W. Complete the following equations (where u and v are vectors in V and k is any scalar):
   a. T (u + v) =
   b. T (ku) =

   Now state the three properties of the kernel and the image of a transformation, and relate these to the notion of subspace.

2. Consider $T : \mathbf{R}^3 \rightarrow \mathbf{R}^3$ a matrix transformation given by:

$$T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 4 \\ 3 & 4 & 7 \\ -2 & 2 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Show that the Kernel of T is a line through the origin, and find parametric equations of that line.

3. In $\mathbf{R}^2$ let l be a line through the origin that makes an amgle of 60° with the positive x-axis. Let a transformation $T : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ be the shear along the line l of magnitude 2. Find the matrix A corresponding to the linear transformation T.

4. Given $v_1 = (2, 3, 1)$, $v_2 = (-1, -2, 3)$, and $v_3 = (10, 17, -9)$, show that v_3 can be written as a linear combination of the other 2 vectors.

5. The points (-2, 7); (12, 5), and (4, -11) are points of a circle whose equation has the form $x^2 + y^2 + bx + cy + d = 0$. Write the linear system that can be used to find b, c, and d. After finding the system, show the augmented matrix you would use to solve the system, the reduced row echelon form obtained from this augmented matrix, and the actual solutions.