1.1/10, 14, 20, 24, 26, 28 Solutions

10) $\left.\begin{aligned} x + 2y + 3z &= 1 \\ 2x + 4y + 7z &= 2 \\ 3x + 7y + 11z &= 8 \end{aligned}\right\} \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 2 & 4 & 7 & 2 \\ 3 & 7 & 11 & 8 \end{array}\right] \begin{matrix} \\ \text{II}-2\text{I} \Rightarrow \\ \text{III}-2\text{I} \end{matrix} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 2 & 5 \end{array}\right] \begin{matrix} x+2y+3z=1 \Rightarrow x=-9 \\ \Rightarrow z=0 \\ y+2z=5 \Rightarrow y=5 \end{matrix}$

solution: $\boxed{(-9, 5, 0)}$

14) $\left.\begin{aligned} x + 4y + z &= 0 \\ 4x + 13y + 7z &= 0 \\ 7x + 22y + 13z &= 1 \end{aligned}\right\} \Rightarrow \left[\begin{array}{ccc|c} 1 & 4 & 1 & 0 \\ 4 & 13 & 7 & 0 \\ 7 & 22 & 13 & 1 \end{array}\right] \begin{matrix} \\ \text{II}-4\text{I} \Rightarrow \\ \text{III}-7\text{I} \end{matrix} \left[\begin{array}{ccc|c} 1 & 4 & 1 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & -6 & 6 & 1 \end{array}\right]$

$\begin{matrix} -\frac{1}{3}\text{II} \Rightarrow \\ \text{III}-2\text{II} \end{matrix} \left[\begin{array}{ccc|c} 1 & 4 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$ ← this means $0x+0y+0z=1$, inconsistent, so no solution. 3 planes never intersect

20) a = output of industry A

b = output of industry B

$\Rightarrow \left.\begin{aligned} a &= 1000 + .1b \\ b &= 780 + .2a \end{aligned}\right\} \Rightarrow \left.\begin{aligned} a - .1b &= 1000 \\ -.2a - b &= 780 \end{aligned}\right\}$

$\Rightarrow$ [redacted] $\left[\begin{array}{cc|c} 1 & -.1 & 1000 \\ -.2 & 1 & 780 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & -.1 & 1000 \\ 0 & .98 & 980 \end{array}\right] \Rightarrow \boxed{\begin{aligned} a &= 1100 \\ b &= 1000 \end{aligned}}$

24) b = rate of boat

r = rate of river

rate downstream = $b + r = \frac{8\text{km}}{20\text{min}} = \frac{2}{5}$

rate upstream = $b - r = \frac{8\text{km}}{40\text{min}} = \frac{1}{5}$

$\Rightarrow \left[\begin{array}{cc|c} b & r & \\ 1 & 1 & \frac{2}{5} \\ 1 & -1 & \frac{1}{5} \end{array}\right] \Rightarrow \text{II}-\text{I} \left[\begin{array}{cc|c} 1 & 1 & \frac{2}{5} \\ 0 & -2 & -\frac{1}{5} \end{array}\right] \Rightarrow \boxed{\begin{aligned} b &= \tfrac{3}{10}\ \text{km/min} \\ r &= \tfrac{1}{10}\ \text{km/min} \end{aligned}}$

26) $\begin{bmatrix} x+y- & z=2 \\ x+2y+ & z=3 \\ x+y+(k^2-5)z & =k \end{bmatrix} \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 1 & 2 & 1 & 3 \\ 1 & 1 & (k^2-5) & k \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & k^2-4 & k-2 \end{array}\right]$

$\Rightarrow (k^2-4)z = k-2 \Rightarrow (k-2)(k+2)z = k-2$

i) if $k=2 \Rightarrow (2-2)(2+2)(z) = 2-2 \Rightarrow 0z=0$ true for all z

$\Rightarrow$ infinitely many solutions

ii) if $k=-2 \Rightarrow (-2-2)(-2+2)z = (-2-2) \Rightarrow 0z=-4$ never true

$\Rightarrow$ no solutions

iii) if $k \neq 2, -2 \Rightarrow z = \dfrac{k-2}{(k-2)(k+2)} = \dfrac{1}{k+2}$ so unique z allows us to solve uniquely for x, y.

$\Rightarrow$ unique solution

28) $\left.\begin{aligned} T_2 &= \frac{T_3+T_1+200+0}{4} \\ T_1 &= \frac{0+0+200+T_2}{4} \\ T_3 &= \frac{T_2+0+0+400}{4} \end{aligned}\right\} \Rightarrow \begin{aligned} T_1 - 4T_2 + T_3 &= -200 \\ -4T_1 + T_2 + 0 &= -200 \\ 0 + T_2 - 4T_3 &= -400 \end{aligned}$

$\Rightarrow \left[\begin{array}{ccc|c} 1 & -4 & 1 & -200 \\ -4 & 1 & 0 & -200 \\ 0 & 1 & -4 & -400 \end{array}\right] \xrightarrow{II-4I} \left[\begin{array}{ccc|c} 1 & -4 & 1 & -200 \\ 0 & -15 & 4 & -1000 \\ 0 & 1 & -4 & -400 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & -4 & 1 & -200 \\ 0 & -15 & 4 & -1000 \\ 0 & 0 & -56 & -7000 \end{array}\right]$ $15III-II$

$\Rightarrow \begin{aligned} T_1 &= 75 \\ T_2 &= 100 \\ T_3 &= 125 \end{aligned} \quad \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 75 \\ 100 \\ 125 \end{bmatrix}$