1.2/ 6, 11, 20, 30, 32, 38

Solutions

6) $\left.\begin{aligned} x_1 - 7x_2 \quad\quad + x_5 &= 3 \\ x_3 \quad -2x_5 &= 2 \\ x_4 + x_5 &= 1 \end{aligned}\right\} \Rightarrow \left[\begin{array}{ccccc|c} 1 & -7 & 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 0 & -2 & 2 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{array}\right]$ ← RREF $\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 3+7s-t \\ s \\ 2+2t \\ 1-t \\ t \end{bmatrix}$

11) $\left.\begin{aligned} x_1 + 2x_3 + 4x_4 &= -8 \\ x_2 + -3x_3 - x_4 &= 6 \\ 3x_1 + 4x_2 - 6x_3 + 8x_4 &= 0 \\ -x_2 + 3x_3 + 4x_4 &= -12 \end{aligned}\right\} \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 2 & 4 & -8 \\ 0 & 1 & -3 & -1 & 6 \\ 3 & 4 & -6 & 8 & 0 \\ 0 & -1 & 3 & 4 & -12 \end{array}\right] \xRightarrow[R_4+R_2]{R_3-3R_1} \begin{bmatrix} 1 & 0 & 2 & 4 & -8 \\ 0 & 1 & -3 & -1 & 6 \\ 0 & 4 & -12 & -4 & 24 \\ 0 & 0 & 0 & 3 & -6 \end{bmatrix}$

$\xRightarrow[\frac{1}{3}R_4]{R_3-4R_2} \begin{bmatrix} 1 & 0 & 2 & 4 & -8 \\ 0 & 1 & -3 & -1 & 6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix} \xRightarrow[\text{swap}(R_3,R_4)]{R_1-4R_4,\ R_2+R_4}$ RREF $\left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 4 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2t \\ 4+3t \\ t \\ -2 \end{bmatrix}$

20) 4

$\begin{bmatrix} 1 & * \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $*$ = any number

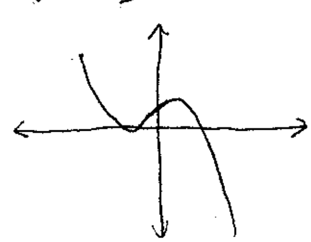
30) $f(t) = a + bt + ct^2 + dt^3$

Point	
(0,1)	$1 = a$
(1,0)	$0 = a + b + c + d$
(-1,0)	$0 = a - b + c - d$
(2,-15)	$-15 = a + 2b + 4c + 8d$

$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 1 & -1 & 0 \\ 1 & 2 & 4 & 8 & -15 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & -1 \\ 0 & -1 & 1 & -1 & -1 \\ 0 & 2 & 4 & 8 & -16 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & -1 \\ 0 & 0 & 2 & 0 & -2 \\ 0 & 0 & 2 & 6 & -14 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 6 & -12 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix} \Rightarrow$

$f(t) = 1 + 2t - t^2 - 2t^3$



32) a) The 1st condition, that $f_i'(a_i) = f_{i+1}'(a_i)$ ensures that the slope of the track does not suddenly jump from one value to ~~etho~~ another. There won't be sharp angles.
The 2nd condition, $f_i''(a_i) = f_{i+1}''(a_i)$, ensures that the slope changes smoothly along the track.
$f_1'(a_0) = f_n'(a_n) = 0$ allows the riders to get on & off the roller coaster in a horizontal position.

b) # of variables $= 4n$ since there are n ~~equatio~~ polynomials, each with 4 coefficients

c) # of equations $= 4n$

requirement:	# equations:
$f_i(a_i) = b_i$	n
$f_i(a_{i-1}) = b_{i-1}$	n
$f_i'(a_i) = f_{i+1}'(a_i)$	$n-1$
$f''(a_i) = f_{(i+1)}''(a_i)$	$n-1$
$f_1'(a_0) = f_n'(a_n) = 0$	2 +
total	$4n$

38)
a) consumer demand $\vec{b} = \begin{bmatrix} 320 \\ 90 \\ 150 \end{bmatrix}$ $\vec{V}_1 = \begin{bmatrix} 0 \\ .1 \\ .2 \end{bmatrix}$ $\vec{V}_2 = \begin{bmatrix} .2 \\ 0 \\ .5 \end{bmatrix}$ $\vec{V}_3 = \begin{bmatrix} .3 \\ .4 \\ 0 \end{bmatrix}$

b) $x_j \vec{V}_j$ gives the demand industry j has on its own product for each ~~#1~~ industry j produces.

c) $x_1\vec{V}_1 + x_2\vec{V}_2 + \dots + x_n\vec{V}_n + \vec{b}$ = the sum of industry demand ~~on~~ and consumer demand.

d) $x_1\vec{V}_1 + x_2\vec{V}_2 + \dots + x_n\vec{V}_n + \vec{b} = \vec{x}$ means the sum of industry demand & consumer demand equals total output.