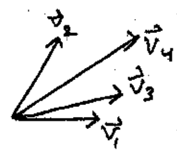
1.3/4, 8, 14, 34, 48, 50 Solutions

4) $\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} \Rightarrow \Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ RREF $\therefore \boxed{\text{rank} = 2}$

8) The system $x\vec{v}_1 + y\vec{v}_2 + z\vec{v}_3 = \vec{v}_4$ could have infinitely many solutions. This is because you only need two vectors to ~~define~~ describe any other vector in 2D (example is $\hat{\imath}, \hat{\jmath}$ unit vectors). So for any value of $z \Rightarrow x\vec{v}_1 + y\vec{v}_2 = (\vec{v}_4 - z\vec{v}_3)$ so we can find an x & a y which satisfies the requirement.



14)

a) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} = -1\begin{bmatrix} 1 \\ 2 \end{bmatrix} + 2\begin{bmatrix} 2 \\ 3 \end{bmatrix} + 1\begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -1\cdot 1 + 2\cdot 2 + 3\cdot 1 \\ -1\cdot 2 + 3\cdot 2 + 4\cdot 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$

34)

a) $A\vec{e}_1 = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 1\begin{bmatrix} a \\ d \\ g \end{bmatrix} + 0\begin{bmatrix} b \\ e \\ h \end{bmatrix} + 0\begin{bmatrix} c \\ f \\ k \end{bmatrix} = \begin{bmatrix} a \\ d \\ g \end{bmatrix}$

$A\vec{e}_2 = \begin{bmatrix} b \\ e \\ h \end{bmatrix}$ $\quad A\vec{e}_3 = \begin{bmatrix} c \\ f \\ k \end{bmatrix}$

b) B is an $m \times 3$ matrix $\Rightarrow B = \begin{bmatrix} | & | & | \\ \vec{v}_1 & \vec{v}_2 & \vec{v}_3 \\ | & | & | \end{bmatrix}$

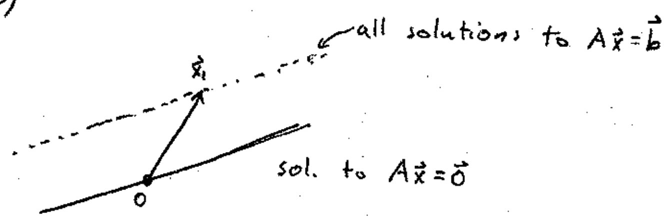
$\Rightarrow B\vec{e}_1 = \begin{bmatrix} | & | & | \\ \vec{v}_1 & \vec{v}_2 & \vec{v}_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 1\begin{bmatrix} | \\ \vec{v}_1 \\ | \end{bmatrix} + 0\vec{v}_2 + 0\vec{v}_3 = \vec{v}_1$

$B\vec{e}_2 = \vec{v}_2$; $B\vec{e}_3 = \vec{v}_3$

48) a) $A(\vec{x}_1+\vec{x}_h) = A\vec{x}_1 + A\vec{x}_h = \vec{b}+\vec{0} = \vec{b}$

b) $A(\vec{x}_2-\vec{x}_1) = A\vec{x}_2 - A\vec{x}_1 = \vec{b}-\vec{b} = \vec{0}$

c)



50) $A\vec{x}=\vec{b}$

A is a 4 x3 matrix. This means there can be a max of 3 leading ones, since no more than one leading one is allowed per column.

$\Rightarrow$ rank$[A] \leq 3$

but rank of augmented matrix is 4 (rank$[A\vdots\vec{b}]=4$)

$\Rightarrow$ no solutions since rank$[A\vdots\vec{b}] >$ rank$[A]$