§2.1 pp. 60–64 #4, 8, 10, 20, 28, 42 Bryant Mathews

Due Wednesday, February 16

(4)
$$\begin{pmatrix} 9 & 3 & -3 \\ 2 & -9 & 1 \\ 4 & -9 & -2 \\ 5 & 1 & 5 \end{pmatrix}$$

(8)
$$\begin{pmatrix} x_1 + 7x_2 = y_1 \\ 3x_1 + 20x_2 = y_2 \end{pmatrix} \underset{-3(\mathrm{I})}{\rightsquigarrow} \begin{pmatrix} x_1 + 7x_2 = y_1 \\ -x_2 = y_2 - 3y_1 \end{pmatrix}$$

$$\underset{\times(-1)}{\rightsquigarrow} \begin{pmatrix} x_1 + 7x_2 = y_1 \\ x_2 = 3y_1 - y_2 \end{pmatrix} \overset{-7(\mathrm{II})}{\rightsquigarrow} \begin{pmatrix} x_1 = -20y_1 + 7y_2 \\ x_2 = 3y_1 - y_2 \end{pmatrix}.$$

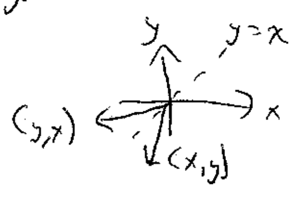
(10)
$$\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 4 & 9 & 0 & 1 \end{array}\right) \underset{-4(\mathrm{I})}{\rightsquigarrow} \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & -4 & 1 \end{array}\right)$$

$$\overset{-2(\mathrm{II})}{\rightsquigarrow} \left(\begin{array}{cc|cc} 1 & 0 & 9 & -2 \\ 0 & 1 & -4 & 1 \end{array}\right), \text{ so } \begin{pmatrix} 1 & 2 \\ 4 & 9 \end{pmatrix}^{-1} = \begin{pmatrix} 9 & -2 \\ -4 & 1 \end{pmatrix}.$$

(20)
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \text{ so}$$

if we let $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, then $A(\vec{e}_1) = \vec{e}_2$, $A(\vec{e}_2) = \vec{e}_1$.

Thus A maps (x,y) to (y,x), or reflects about the line $y = x$.

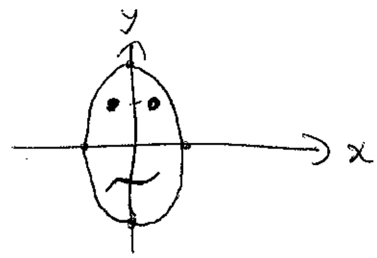


(20 cont.)

You can check that $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}^{-1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

This makes sense geometrically: if you reflect twice about the same line, that is the same as the identity transformation.

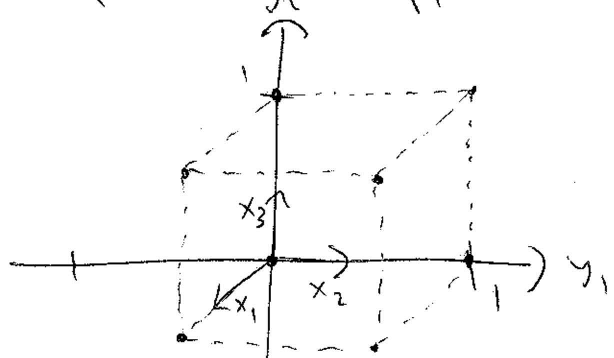
28 $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ this stretches by a factor of 2 in the y-direction.



42 a) Let $A = \begin{pmatrix} -\frac{1}{2} & 1 & 0 \\ -\frac{1}{2} & 0 & 1 \end{pmatrix}$. Then

$$A\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad A\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}, \quad A\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad A\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$A\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}, \quad A\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad A\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix}, \quad A\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}.$$



b) $\begin{cases} -\frac{1}{2}x_1 + x_2 = 0 \\ -\frac{1}{2}x_1 + x_3 = 0 \end{cases}$

We can parametrize the solutions by $(2t, t, t)$, $t \in \mathbb{R}$.