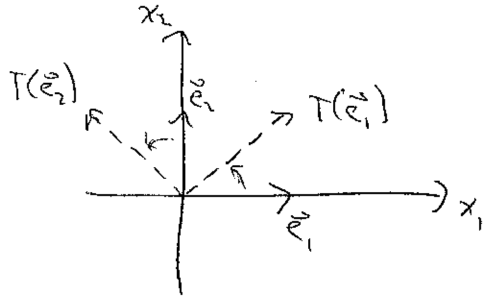
§2.2 pp. 77–84 # 4, 8, 10, 32, 42, 50

Bryant Mathews

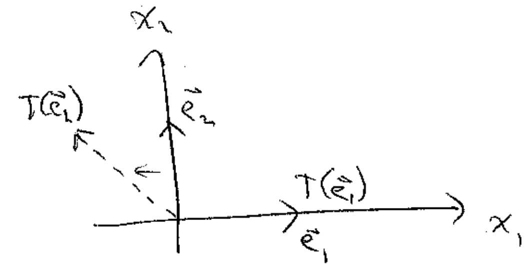
Due Friday, Feb. 18

(4) $T(\vec{x}) = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \vec{x}$



rotation by $\frac{\pi}{4}$ and dilation by factor of $\sqrt{2}$.

(8) $T(\vec{x}) = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \vec{x}$



a shear parallel to the x_1-axis.

(10) The unit vector $\vec{u} = \begin{pmatrix} 4/5 \\ 3/5 \end{pmatrix}$ points in the direction of the line L. Let $\vec{w}$ be the projection of $\vec{v}$ onto L. Then $\vec{w} = (\vec{u} \cdot \vec{v})\, \vec{u}$,

or

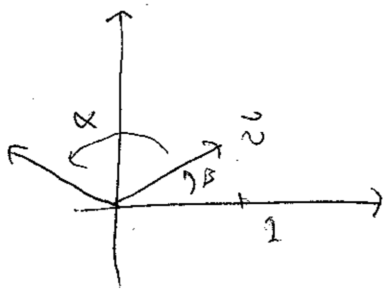
$$\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \left(\begin{pmatrix} 4/5 \\ 3/5 \end{pmatrix} \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \right) \begin{pmatrix} 4/5 \\ 3/5 \end{pmatrix}$$

$$= \left(\frac{4}{5} v_1 + \frac{3}{5} v_2 \right) \begin{pmatrix} 4/5 \\ 3/5 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{16}{25} v_1 + \frac{12}{25} v_2 \\ \frac{12}{25} v_1 + \frac{9}{25} v_2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{16}{25} & \frac{12}{25} \\ \frac{12}{25} & \frac{9}{25} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}.$$

(32) a) $D\vec{v}$



b) $$D\vec{v} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} \cos\beta \\ \sin\beta \end{pmatrix}$$

$$= \begin{pmatrix} \cos\alpha\cos\beta - \sin\alpha\sin\beta \\ \sin\alpha\cos\beta + \cos\alpha\sin\beta \end{pmatrix}$$

(which equals $\begin{pmatrix} \cos(\alpha+\beta) \\ \sin(\alpha+\beta) \end{pmatrix}$, by part a).

(47) a) Let $T = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Then

$$f(t) = \left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} \cos t \\ \sin t \end{pmatrix}\right) \cdot \left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} -\sin t \\ \cos t \end{pmatrix}\right)$$

$$= (a\cos t + b\sin t)(-a\sin t + b\cos t) + (c\cos t + d\sin t)(-c\sin t + d\cos t),$$

which is built by addition and multiplication from $\sin t$, $\cos t$, and constant functions, all continuous.

b) $$f\left(\tfrac{\pi}{2}\right) = \left(T\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) \cdot \left(T\begin{pmatrix} -1 \\ 0 \end{pmatrix}\right) = \left(T\begin{pmatrix} -1 \\ 0 \end{pmatrix}\right) \cdot \left(T\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) = -\left(T\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right) \cdot \left(T\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right)$$

$$= -f(0).$$

(47 cont.)

c) We apply the Intermediate value theorem.

Here, $g(t) = f(t)$, $a = 0$, $b = \frac{\pi}{2}$, $L = 0$ (which must necessarily lie between $f(a)$ and $f(b) = -f(a)$).

The theorem guarantees the existence of a c s.t. $0 = a \leq c \leq b = \frac{\pi}{2}$ with $f(c) = 0$.

d) Let $\vec{v}_1 = \begin{pmatrix} \cos c \\ \sin c \end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix} -\sin c \\ \cos c \end{pmatrix}$, using c from part (c).

We know that $\vec{v}_1 \cdot \vec{v}_2 = -\cos c \sin c + \sin c \cos c = 0$.

Also, $f(c) = 0$, where

$$f(c) = T(\vec{v}_1) \cdot T(\vec{v}_2), \text{ so}$$

$T(\vec{v}_1)$ and $T(\vec{v}_2)$ are also perpendicular.

50 Using the hint we parametrize the unit circle with $\vec{v}(t) = \cos(t)\, \vec{v}_1 + \sin(t)\, \vec{v}_2$.

Applying T, we have

$$\begin{aligned} T\vec{v}(t) &= T\left(\cos(t)\, \vec{v}_1 + \sin(t)\, \vec{v}_2\right) \\ &= \cos(t)\, T\vec{v}_1 + \sin(t)\, T\vec{v}_2 \\ &= \cos(t)\, \vec{w}_1 + \sin(t)\, \vec{w}_2, \end{aligned}$$

with w_1, w_2 perpendicular. By the footnote, this is an ellipse.