Homework 2.4 : 4, 14, 26, 28, 40, 76

4. $$\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$$

Note: Since the first matrix is a 3×2 matrix and the second is a 2×2 matrix, we can multiply them.

$$= \begin{bmatrix} 1(3)-1(1) & 1(2)-1(0) \\ 0(3)+2(1) & 0(2)+2(0) \\ 2(3)+1(1) & 2(2)+1(0) \end{bmatrix} = \begin{bmatrix} 3-1 & 2-0 \\ 0+2 & 0+0 \\ 6+1 & 4+1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 0 \\ 7 & 5 \end{bmatrix}$$

14. A is 2×2, B is 1×3, C is 3×3, D is 3×1, and E is 1×1

Therefore the valid products are AA, CC, EE, BC, BD, DB, EB, CD, DE.

$$AA = \begin{bmatrix} 1(1)+1(1) & 1(1)+1(1) \\ 1(1)+1(1) & 1(1)+1(1) \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$CC = \begin{bmatrix} 1(1)+0(2)-1(3) & 1(0)+0(1)-1(2) & 1(-1)+0(0)-1(1) \\ 2(1)+1(2)+0(3) & 2(0)+1(1)+0(2) & 2(-1)+1(0)+0(1) \\ 3(1)+2(2)+1(3) & 3(0)+2(1)+1(2) & 3(-1)+2(0)+1(1) \end{bmatrix} = \begin{bmatrix} -2 & -2 & -2 \\ 4 & 1 & -2 \\ 10 & 4 & -2 \end{bmatrix}$$

$$EE = \begin{bmatrix} 5(5) \end{bmatrix} = \begin{bmatrix} 25 \end{bmatrix}$$

$$BC = \begin{bmatrix} 1(1)+2(2)+3(3) & 1(0)+2(1)+3(2) & 1(-1)+2(0)+3(1) \end{bmatrix} = \begin{bmatrix} 14 & 8 & 2 \end{bmatrix}$$

$$BD = \begin{bmatrix} 1(1)+2(1)+3(1) \end{bmatrix} = \begin{bmatrix} 6 \end{bmatrix}$$

$$DB = \begin{bmatrix} 1(1) & 1(2) & 1(3) \\ 1(1) & 1(2) & 1(3) \\ 1(1) & 1(2) & 1(3) \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$$

$$EB = \begin{bmatrix} 5(1) & 5(2) & 5(3) \end{bmatrix} = \begin{bmatrix} 5 & 10 & 15 \end{bmatrix}$$

$$CD = \begin{bmatrix} 1(1)+0(1)-1(1) \\ 2(1)+1(1)+0(1) \\ 3(1)+2(1)+1(1) \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix}$$

$$DE = \begin{bmatrix} 1(5) \\ 1(5) \\ 1(5) \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}$$

26. $\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}\begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix} = \begin{bmatrix} A_1B_1 + A_2B_3 & A_1B_2 + A_2B_4 \\ A_3B_1 + A_4B_3 & A_3B_2 + A_4B_4 \end{bmatrix}$

$A_1B_1 + A_2B_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$A_1B_2 + A_2B_4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix}$

$A_3B_1 + A_4B_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$A_3B_2 + A_4B_4 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$\Rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]\left[\begin{array}{cc|cc} 1 & 2 & 2 & 3 \\ 3 & 4 & 4 & 5 \\ \hline 0 & 0 & 1 & 2 \\ 0 & 0 & 3 & 4 \end{array}\right] = \left[\begin{array}{cc|cc} 1 & 2 & 3 & 5 \\ 3 & 4 & 7 & 9 \\ \hline 0 & 0 & 1 & 2 \\ 0 & 0 & 3 & 4 \end{array}\right]$ (longhand multiplication yields the same result)

28. $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\quad A^2 = \begin{bmatrix} a^2+bc & b(a+d) \\ c(a+d) & bc+d^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Trying values that satisfy $a+d=0$ and $a^2+bc=0$, we find the matrix $A = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$ which satisfies $A^2 = 0$.
(there are other possibilities as well).

40. $B^{-1}A^{-1} = (AB)^{-1}$. Let's say that $A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Then...

$\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a+2c & b+2d \\ 3a+5c & 3b+5d \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$

So we want to solve the system: $a+2c=1$ $\quad b+2d=3$
$3a+5c=2$ $\quad 3b+5d=5$

This yields $a=-1$, $b=-5$, $c=1$, $d=4$. So $A^{-1} = \begin{bmatrix} -1 & -5 \\ 1 & 4 \end{bmatrix}$

Then $A = \frac{1}{-1(4)-1(-5)}\begin{bmatrix} 4 & 5 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ -1 & -1 \end{bmatrix}$

76a. $I = \frac{1}{3}(R+G+B)$
$L = R - G$
$S = B - \frac{1}{2}(R+G)$

So the matrix P that sends $\begin{bmatrix} R \\ G \\ B \end{bmatrix}$ to $\begin{bmatrix} I \\ L \\ S \end{bmatrix}$ is..

$$P = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 1 & -1 & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 1 \end{bmatrix}$$

b. If the sunglasses cut off all the blue light but don't affect the red or green light, then

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

c. The matrix representing what light does when it goes through sunglasses and then the eye is

$$PA = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & 0 \\ 1 & -1 & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 0 \end{bmatrix}$$

d. Putting on the sunglasses is like first transforming $\begin{bmatrix} R \\ G \\ B \end{bmatrix}$ to $\begin{bmatrix} I \\ L \\ S \end{bmatrix}$ and then undergoing transformation M. But this has the exact same result as if you were wearing the glasses and viewed the light. So we get $MP = PA \Rightarrow M = PAP^{-1}$.

Therefore, $M = \begin{bmatrix} \frac{2}{3} & 0 & -\frac{2}{9} \\ 0 & 1 & 0 \\ -1 & 0 & \frac{1}{3} \end{bmatrix}$