Solutions to section 3.3

22 A basis for the kernel is $\left\{\begin{pmatrix}1\\-1\\0\\0\\0\end{pmatrix}, \begin{pmatrix}3\\9\\-1\\-1\\0\end{pmatrix}\right\}$ dim ker $=2$

A basis for the image is $\left\{\begin{pmatrix}1\\1\\2\\0\end{pmatrix}, \begin{pmatrix}1\\2\\3\\1\end{pmatrix}, \begin{pmatrix}1\\2\\3\\-1\end{pmatrix}\right\}$ dim im $=3$

$2+3=5$, and the matrix is 4×5, so it works.

32 We want $\vec{v}$ s.t. $v \cdot \begin{pmatrix}1\\0\\-1\\1\end{pmatrix} = v \cdot \begin{pmatrix}0\\1\\2\\3\end{pmatrix} = 0$. If $v = \begin{pmatrix}v_1\\v_2\\v_3\\v_4\end{pmatrix}$, we get

$$\begin{aligned} v_1 \quad - v_3 + v_4 &= 0 \\ v_2 + 2v_3 + 3v_4 &= 0. \end{aligned}$$

This is already rref, so two LI v's to form the basis are

$\begin{pmatrix}1\\-2\\1\\0\end{pmatrix}$ and $\begin{pmatrix}-1\\-3\\1\\0\end{pmatrix}$.

36 If $\mathrm{im}(A) = \ker(A)$, then $\underset{3=}{}\dim \mathrm{im}(A) + \dim \ker(A) = 2 \cdot \dim \mathrm{im}(A)$, so $\dim \mathrm{im}(A) = 1.5$. This makes no sense, so we can't find such a matrix.

40 Call $\dim V = n$. If $V \subseteq W$, then we can write an n-vector basis for V and all these vectors must be in W and must still be LI in W. So we can find n LI vectors in W, so $\dim W \geq n$, so $\dim W \geq \dim V$.

50 A matrix in rref necessarily has all its non-zero rows LI, because if they weren't we could reduce one to zero. So the non-zero rows of an rref matrix form a basis for row space. Since every leading 1 in an rref matrix corresponds to a basis vector for the image and every non-zero row has a leading 1, dim (row space) = rank (E).

56 Well, the hint pretty much is the answer here. If we take $c_0\vec{v} + \ldots + c_{m-1}A^{m-1}\vec{v} = \vec{0}$ and multiply both

sides by A^{m-1}, we get $c_0 A^{m-1}\vec{v}=0$, since all other terms have an A^m and thus die. We're given $A^{m-1}\vec{v}\neq 0$, so $c_0=0$. With that, we take the original eqn. and multiply by A^{m-2} to show $c_1=0$, and so on down the line. So all the c's are 0, so $\vec{v}, A\vec{v}, \ldots, A^{m-1}\vec{v}$ are LI.