Solutions to section 4.1

6 $\|u\| = \sqrt{1^2+1^2+2^2+2^2} = \sqrt{10}$

$\|v\| = \sqrt{2^2+3^2+4^2+5^2} = \sqrt{54} = 3\sqrt{6}$

$u \cdot v = 1\times 2 - 1\times 3 + 2\times 4 - 2\times 5 = -3$

$\frac{u\cdot v}{\|u\|\cdot\|v\|} = \frac{-1}{\sqrt{60}} = \frac{-1}{2\sqrt{15}} = \cos\theta$

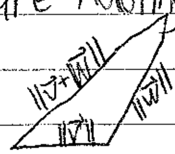
$\theta = \cos^{-1}\left(\frac{-1}{2\sqrt{15}}\right) \approx 1.7$ radians

12 $\|\vec{v}+\vec{w}\|^2 = (\vec{v}+\vec{w})\cdot(\vec{v}+\vec{w}) = \|\vec{v}\|^2 + \|\vec{w}\|^2 + 2\vec{v}\cdot\vec{w} \leq \|\vec{v}\|^2 + \|\vec{w}\|^2 + 2|\vec{v}\cdot\vec{w}|$

$\leq \|\vec{v}\|^2 + \|\vec{w}\|^2 + 2\|\vec{v}\|\|\vec{w}\|$ by Cauchy-Schwartz

$\leq (\|\vec{v}\| + \|\vec{w}\|)^2$

Square rooting, $\|\vec{v}+\vec{w}\| \leq \|\vec{v}\| + \|\vec{w}\|$



14 ~~FA (F₂ sin α) k, where k is an arbitrary constant~~

$\|F_1\| \sin\beta = \|F_2\| \sin\alpha$ so that horizontal force is equal.

By trig, $\frac{BE}{ED} = \tan\beta$ and $\frac{AE}{ED} = \tan\alpha$

So $\frac{AE}{BE} = \frac{\tan\alpha}{\tan\beta}$.

But $\frac{\|F_1\|}{\|F_2\|} = \frac{\sin\alpha}{\sin\beta} \neq \frac{\tan\alpha}{\tan\beta}$, so Leonardo was wrong.

16 All we can find is $\vec{v_4} = \begin{pmatrix} -1/2 \\ 1/2 \\ 1/2 \\ -1/2 \end{pmatrix}$ and $-\vec{v_4} = \begin{pmatrix} 1/2 \\ -1/2 \\ -1/2 \\ 1/2 \end{pmatrix}$ — both work.

26 $\begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -6 \\ 2 \end{pmatrix}$ are $\perp$ (check w/ dot product), so we only have to normalize them.

$\left\|\begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}\right\| = \left\|\begin{pmatrix} 3 \\ -6 \\ 2 \end{pmatrix}\right\| = \sqrt{3^2+6^2+2^2} = 7$

So let $\vec{v_1} = \begin{pmatrix} 2/7 \\ 3/7 \\ 6/7 \end{pmatrix}$, $\vec{v_2} = \begin{pmatrix} 3/7 \\ -6/7 \\ 2/7 \end{pmatrix}$

$\text{Proj}_{v_1}\begin{pmatrix} 49 \\ 49 \\ 49 \end{pmatrix} = \left(\frac{2}{7}\times 49 + \frac{3}{7}\times 49 + \frac{6}{7}\times 49\right)\begin{pmatrix} 2/7 \\ 3/7 \\ 6/7 \end{pmatrix} = \begin{pmatrix} 22 \\ 33 \\ 66 \end{pmatrix}$

$\text{Proj}_{v_2}\begin{pmatrix} 49 \\ 49 \\ 49 \end{pmatrix} = \left(\frac{3}{7}\times 49 - \frac{6}{7}\times 49 + \frac{2}{7}\times 49\right)\begin{pmatrix} 3/7 \\ -6/7 \\ 2/7 \end{pmatrix} = \begin{pmatrix} -3 \\ 6 \\ -2 \end{pmatrix}$

Total projection $= \begin{pmatrix} 22 \\ 33 \\ 66 \end{pmatrix} + \begin{pmatrix} -3 \\ 6 \\ -2 \end{pmatrix} = \begin{pmatrix} 19 \\ 39 \\ 64 \end{pmatrix}$

32 $\det G = (\vec{v}_1 \cdot \vec{v}_1)(\vec{v}_2 \cdot \vec{v}_2) - (\vec{v}_1 \cdot \vec{v}_2)^2$

$= \|\vec{v}_1\|^2 \|\vec{v}_2\|^2 - (\vec{v}_1 \cdot \vec{v}_2)^2$

If $\det G = 0$, $\|\vec{v}_1\|^2 \|\vec{v}_2\|^2 = (\vec{v}_1 \cdot \vec{v}_2)^2$

$\|\vec{v}_1\| \, \|\vec{v}_2\| = |\vec{v}_1 \cdot \vec{v}_2|$ which Cauchy-Schwartz says is true iff $\vec{v}_1$ and $\vec{v}_2$ are parallel.

So $\det G \neq 0$ iff $\vec{v}_1$ and $\vec{v}_2$ <u>aren't</u> parallel.