Math 21b Solution Set Section 5.1

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Question 16

There is only one non-zero pattern, i.e. one pattern without zero in any of the factors. This is the following:

$$\begin{bmatrix} \dots & \dots & 2 & \dots & \dots \\ \dots & \dots & \dots & 2 & \dots \\ \dots & 9 & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & 5 \\ 3 & \dots & \dots & \dots & \dots \end{bmatrix}$$

There are four inversions for a_{51} and two inversions for a_{32}, so there are a total of 6 inversions.

Thus, the determinant is $(+1)(3)(9)(2)(2)(5) = 540$.

Question 24

Let $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$ represent the first, second, and third columns of the matrix, respectively. Then we see that $2\vec{v}_2 - \vec{v}_1 = \vec{v}_3$ so that the matrix is not invertible since its columns are linearly dependent, which means that the determinant is 0, by Fact 5.2.5., so we have $\det(A) = 0$.

Question 26

A matrix A is invertible if and if only if $\det(A) = 0$.

Therefore, we need to determine when $\det\left(\begin{bmatrix} \lambda - 1 & -3 \\ 0 & \lambda - 3 \end{bmatrix}\right) = 0$ which is true when $(\lambda - 1)(\lambda - 3) = 0 \Rightarrow \lambda^2 - 4\lambda + 3 = 0 \Rightarrow \lambda = 1$ and 3

Question 36

There is clearly only one pattern that does not contain a 0 as one of its factors, i.e. there is only one pattern that makes a nonzero contribution to $\det(A)$, i.e. the diagonal a_{n1} ... $a_{(n-1)2}$ ... a_{1n}. There are $n-1$ inversions of a_{n1}, and similarly there are $n-1-k$ inversions of the element a_{nk}. Therefore, there are $((n-1)+(n-2)+...+1+0)$ total inversions for this pattern. From elementary algebra, we know what this sum is: $((n-1)+(n-2)+...+1+0)=\frac{(n)(n-1)}{2}$ This sum is even if n or $(n-1)$ is divisible by 4, i.e. if n has a remainder of 0 or 1 when divided by 4. Similarly, this sum is odd if n or $n-1$ is divisible by 2 and *NOT* by 4, i.e. if n has a remainder of 2 or 3 when divided by 4.

Question 48

Using Fact 5.1.2, we see that the determinant is always a sum patterns whic are the product of n entries in an n by n matrix. Therefore, if we consider $\det((k)A)$ in terms of $\det(A)$, we see that we can factor out a k from each entry in a pattern when calculating $\det((k)A)$, and thus we can factor out a k^2 from each pattern. This gives us

$$\det((k)A) = (k^2)\det((k)A)$$

Question 52

The determinant of this matrix is greater than 0.

There is only one pattern that contain all five of the entries equal to 1000, which is below:

$$\begin{bmatrix} ... & 1000 & ... & ... & ... \\ ... & ... & ... & 1000 & ... \\ 1000 & ... & ... & ... & ... \\ ... & ... & ... & ... & 1000 \\ ... & ... & 1000 & ... & ... \end{bmatrix}$$

There are four inversions in this pattern, so that in calculating the determinant the sign is positive.

The remainder of the patterns have at most 3 entries equal to 1000, and the rest of the entries are less than 10. Since the sum of the absolute values of all the other patterns is less than $(5!)(10)(1000)^3 = (1200)(1000)^3$, and the pattern displayed above has a value of 1000^5, so we see that the value of this one positive pattern is significantly greater than the possible sum of all the rest, which means that the sign of the determinant of the matrix is *POSITIVE.*