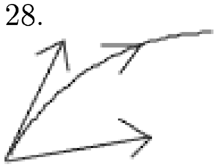
Janak Ramakrishnan

2. Yes, $\vec{v}$ is an eigenvector of A^{-1}. $\vec{v} = A^{-1}A\vec{v} = A^{-1}(\lambda\vec{v}) = \lambda A^{-1}\vec{v}$, so $A^{-1}\vec{v} = \frac{1}{\lambda}\vec{v}$. Its eigenvalue is $\frac{1}{\lambda}$.

14. $A\vec{e}_2 = \vec{a}_2$, where $\vec{a}_2$ is the second column of A. Thus, if $\vec{e}_2$ is an eigenvector of A, $\vec{a}_2$ must be $\lambda\vec{e}_2$ for some λ. Thus, $\vec{a}_2$ must be of the form $(\lambda, 0, 0, \ldots, 0)$. Therefore, any matrix A where $a_{i2} = 0$ for all i (except for 1; if a_{12} is 0, then λ is 0, but it has no restrictions) has $\vec{e}_2$ as an eigenvector.

20. $\vec{e}_3$ is an eigenvector – we are rotating about it, so it doesn't change. Any vector with a component of $\vec{e}_1$ or $\vec{e}_2$ gets rotated, and thus will not be a scalar multiple of itself. So $\vec{e}_3$ is the only eigenvector, with eigenvalue 1.

28.



38.a. $n(t+1) = 2a(t)$, $a(t+1) = a(t) + n(t)$. Thus, $A = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}$

b. $A\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $A\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix} = -1\begin{bmatrix} 2 \\ -1 \end{bmatrix}$

c. $n(0) = 1$, $a(0) = 0$, so the original vector is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$. This vector can be written as $\frac{1}{3}\begin{bmatrix} 1 \\ 1 \end{bmatrix} - \frac{1}{3}\begin{bmatrix} -2 \\ 1 \end{bmatrix}$, so $A^t\begin{bmatrix} 1 \\ 0 \end{bmatrix} = A^t\left(\frac{1}{3}\begin{bmatrix} 1 \\ 1 \end{bmatrix} - \frac{1}{3}\begin{bmatrix} -2 \\ 1 \end{bmatrix}\right) = \frac{1}{3}A^t\begin{bmatrix} 1 \\ 1 \end{bmatrix} - \frac{1}{3}A^t\begin{bmatrix} -2 \\ 1 \end{bmatrix} = \frac{2^t}{3}\begin{bmatrix} 1 \\ 1 \end{bmatrix} - \frac{(-1)^t}{3}\begin{bmatrix} -2 \\ 1 \end{bmatrix}$.

42.a. True. $(\vec{v}\vec{v}^T) \times \vec{v} = \vec{v}\left(\vec{v}^T\vec{v}\right) = \vec{v}\left(\vec{v}\cdot\vec{v}\right) = \left(\vec{v}\cdot\vec{v}\right)\vec{v}$, so $\vec{v}$ is an eigenvector with eigenvalue $\vec{v}\cdot\vec{v} = \|\vec{v}\|^2$.

b. True. $AB\vec{v} = A\lambda_B\vec{v} = \lambda_A\lambda_B\vec{v}$.

c. True. $A\vec{v} = \lambda\vec{v}$. Thus, $A\left(\frac{1}{\lambda}\vec{v}\right) = \vec{v}$, so $\vec{v}$ is in the image. This cannot be done if λ is 0, but then $\vec{v}$ is in the kernel.