6.2 Solutions

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8. One eigenvalue is clearly -3. The other 2 are both 0, since the dimension of the kernel is 2.

10. The characteristic polynomial is $(\lambda+3)(\lambda+1)(\lambda-3)+8(\lambda+1)=\lambda^3+\lambda^2-\lambda-1$. 1 and -1 are clearly factors of this, so λ^2-1 is a factor. We see that $\lambda+1$ is the remaining factor, so -1 has multiplicity 2, and 1 has mult. 1.

16. $Det\,((\lambda I_2-A))=\lambda^2-(a+c)\lambda+ac-b^2$. Thus, $\lambda=\frac{a+c\pm\sqrt{a^2+c^2+2ac-4ac+4b^2}}{2}$. Only if the square root is 0 are the eigenvalues not distinct. This happens when $0=a^2+c^2-2ac+4b^2=(a-c)^2+(2b)^2$, which can occur only if $a=c$ and $b=0$.

20. The determinant is $\lambda_1\lambda_2$, since that is the constant term in the characteristic polynomial by definition, and the determinant is the constant term when we calculate the characteristic polynomial, since every term in the constant term is also a term in the determinant (and vice versa). The factor of -1 cancels, of course. The trace is $\lambda_1+\lambda_2$ since the term multiplying λ will be $-a-c$, which is the negative of the trace, and $-\lambda_1-\lambda_2$ is the actual coefficient. Verified: The characteristic polynomial is $(\lambda-1)(\lambda-3)-8=\lambda^2-4\lambda-5$ and $-5=Det(A)$. $4=Tr(A)$.

28.a. $A=\begin{bmatrix}.8 & .1\\ .2 & .9\end{bmatrix}$. $.8+.2=.9+.1=1$.

b. The eigenvalues and eigenvectors of A are needed. $(\lambda-.8)(\lambda-.9)-.02=\lambda^2-1.7\lambda+.7$, so $\lambda=\frac{1.7\pm\sqrt{2.89-2.8}}{2}=\frac{1.7\pm.3}{2}$, so $\lambda=1$ or $\lambda=.7$. Let $\vec{x}$ and $\vec{y}$ be the eigenvectors. $.8\vec{x}_1+.1\vec{x}_2=x_1$, so $\vec{x}=\begin{bmatrix}1\\2\end{bmatrix}$. $.8\vec{y}_1+.1\vec{y}_2=.7y_1$, so $\vec{y}=\begin{bmatrix}1\\-1\end{bmatrix}$. Then our formula is $400\begin{bmatrix}1\\2\end{bmatrix}+800\times .7^t\begin{bmatrix}1\\-1\end{bmatrix}$.

c. We want to know when the first component is less than 250. The first part of the sum always yields 400. The second part goes to 0, but never goes negative. So the Wipfs will never close down (see 6.4, 30b to understand what is happening here, or just realize that the first vector has eigenvalue 1, and the second has eigenvalue <1, and think about the shape of the phase diagram).

32. The characteristic polynomial is $\lambda^3-3\lambda-k$. Note that the function $\lambda^3-3\lambda-k$ has turning points (critical points) at the roots of $3\lambda^2-3$, or 1 and -1. Thus, we must have that $(-1)^3-3(-1)-k\geq 0$, and $1^3-3(1)-k\leq 0$, so $2\geq k$ and $-2\leq k$, so $-2\leq k\leq 2$ yields three distinct eigenvalues.