6.3 Solutions

14 $\begin{vmatrix} \lambda-1 & 0 & 0 \\ 5 & \lambda & -2 \\ 0 & 0 & \lambda-1 \end{vmatrix} = 0.$ $\qquad \lambda(\lambda-1)^2 = 0$, so $\lambda = 0, 1$

$E_0 = \text{span}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$
$E_1 = \text{span}\left\{\begin{pmatrix} 1 \\ -5 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}\right\}$

22 If $E_7 = \mathbb{R}^2$, all vectors must be scaled by a factor of 7. The only matrix that does this is $\boxed{7I_2}$.

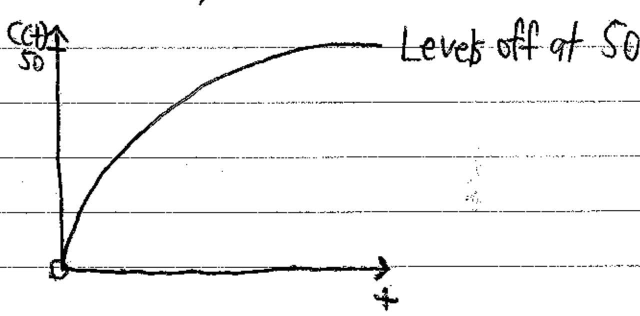
28 Using patterns to find $\det J_n(\lambda)$, we must pick the top left λ, since it's unique to its column. That eliminates the first row, so we pick the second λ and so on, until we find $\boxed{\det J_n(\lambda) = \lambda^n}$

Thus the eigenvalues are 0 with alg. mult. n.
For geom. mult., we just want $\dim \ker \begin{pmatrix} 0 & 1 & & 0 \\ & 0 & \ddots & \\ & & \ddots & 1 \\ 0 & & & 0 \end{pmatrix} = n - \text{rank}$.

But rank $= n-1$ (all columns but the first are clearly LI), so geom. mult. $= 1$, even though alg. mult. $= n$!

38
$C(t+1) = .8C(t) + 10$
So $A = \begin{pmatrix} .8 & 10 \\ 0 & 1 \end{pmatrix}$

Eigenvectors are $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, w/ eigenval. 0.8, and $\begin{pmatrix} 50 \\ 1 \end{pmatrix}$, w/ eigenval. 1.
Given $C(0) = 0$, and $0 = 50 - 50 \times 1$,
$\boxed{C(t) = 50 - 50 \times 0.8^t}$



40a Each week, 20% of the pollutant dissipates, 10% of the pollutant in lake Silv. moves to lake Sils, and 20% of the pollutant in lake Sils moves to lake St.

b Our matrix is lower-triangular, so $\lambda = .7, .6, .8$.

$E_{.8} = \text{span}\begin{pmatrix}0\\0\\1\end{pmatrix}$, $E_{.7} = \text{span}\begin{pmatrix}1\\1\\-2\end{pmatrix}$, $E_{.6} = \text{span}\begin{pmatrix}0\\1\\-1\end{pmatrix}$

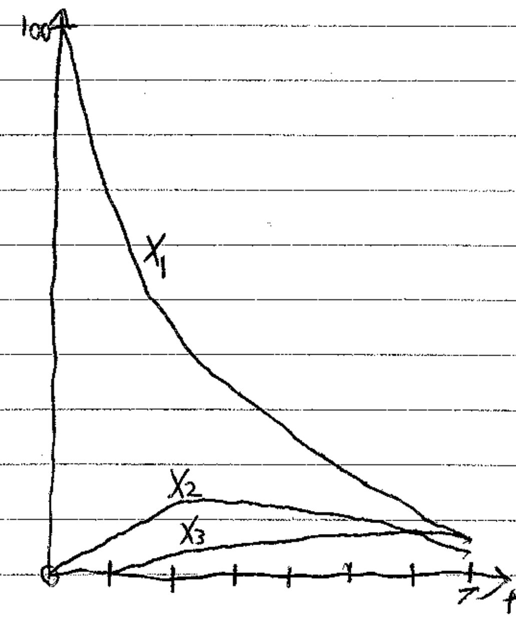
$\vec{x}(0) = \begin{pmatrix}100\\0\\0\end{pmatrix} = 100\begin{pmatrix}1\\1\\-2\end{pmatrix} - 100\begin{pmatrix}0\\1\\-1\end{pmatrix} + 100\begin{pmatrix}0\\0\\1\end{pmatrix}$

So $x_1(t) = 100 \times 0.7^t$

$x_2(t) = 100 \times 0.7^t - 100 \times 0.6^t$

$x_3(t) = 100 \times 0.8^t - 200 \times 0.7^t + 100 \times 0.6^t$

x_2 maxes at 13 on the 2nd day.



44a $a(t+1) = a(t) + j(t)$ $\quad A = \begin{pmatrix}1 & 1\\1 & 0\end{pmatrix}$

$j(t+1) = a(t)$

b $\lambda^2 - \lambda - 1 = 0 \Rightarrow \lambda = \frac{1 \pm \sqrt{5}}{2}$. Call $\lambda_+ = \frac{1+\sqrt{5}}{2}$, $\lambda_- = \frac{1-\sqrt{5}}{2}$.

$E_{\lambda_+} = \text{span}\begin{pmatrix}\lambda_+\\1\end{pmatrix}$, $E_{\lambda_-} = \text{span}\begin{pmatrix}\lambda_-\\1\end{pmatrix}$ — convenient!

$\vec{x}(0) = \begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{\sqrt{5}}\begin{pmatrix}\lambda_+\\1\end{pmatrix} - \frac{1}{\sqrt{5}}\begin{pmatrix}\lambda_-\\1\end{pmatrix}$

$\vec{x}(t) = \frac{1}{\sqrt{5}}\begin{pmatrix}\lambda_+\\1\end{pmatrix}\lambda_+^t - \frac{1}{\sqrt{5}}\begin{pmatrix}\lambda_-\\1\end{pmatrix}\lambda_-^t$

c as $t \to \infty$ λ_-^t term dies, since $|\lambda_-| < 1$.

So $\frac{a(t)}{j(t)} = \frac{\lambda_+^{t+1}}{\lambda_+^t} = \lambda_+ = \boxed{\frac{1+\sqrt{5}}{2}}$