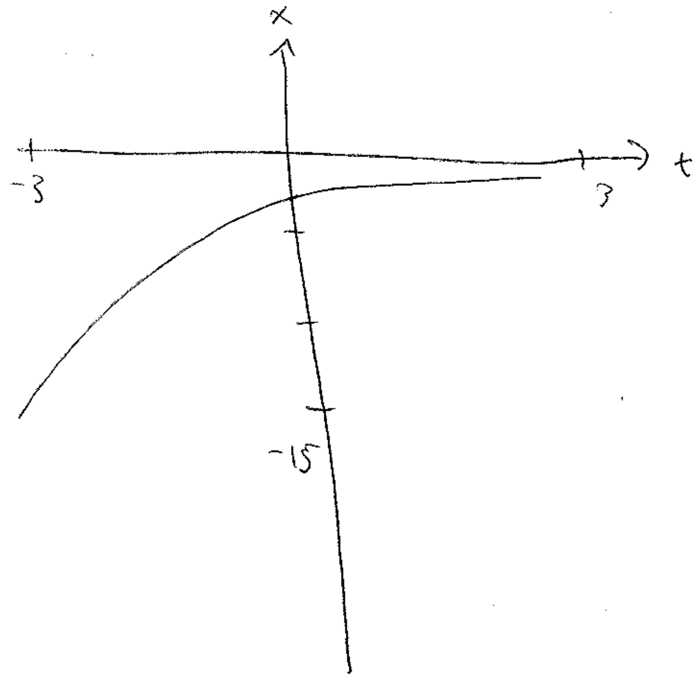
§8.1 pp. 450–457 #2, 14, 26, 32, 46, 54 Bryant Mathews

Due Monday, April 17.

(2) By fact 8.1.1, the solution is

$$x(t) = e^{-.71t}(-e) = -e^{1-.71t}.$$



(14) a) $x(t) = e^{-\frac{1}{8270}t}$.

We want $e^{-\frac{1}{8270}t} = \frac{1}{2}$, or

$$t = -8270 \ln\left(\tfrac{1}{2}\right) \approx 5732 \text{ years.}$$

b) $e^{-\frac{1}{8270}t} = .53$

$\Rightarrow$ $t = -8270 \ln(.53) \approx 5250$ years,

so the man died around 3259 B.C., which is incompatible with the estimate of the Austrian historian.

(26) eigen values: $\lambda_1 = 3$, $\lambda_2 = -2$

eigen vectors: $\vec{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$,

according to Fact 8.1.2,

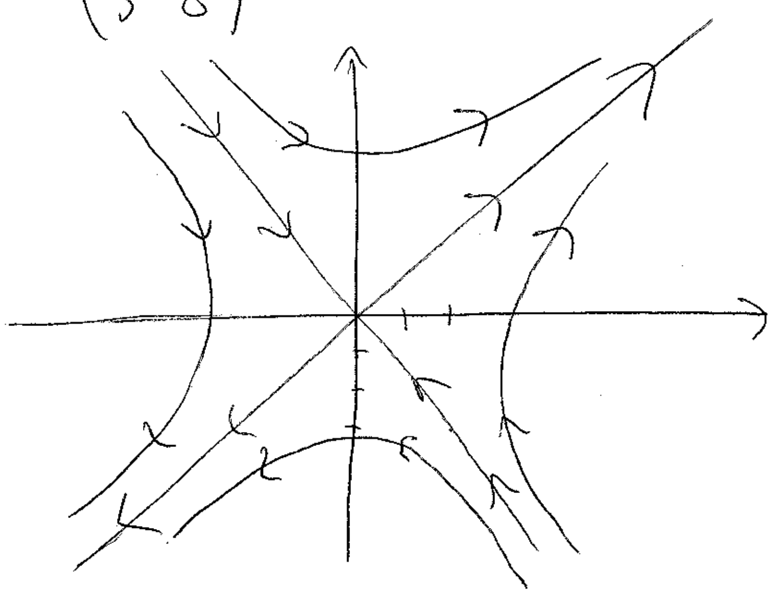
the solution is

$$\vec{x}(t) = 5e^{3t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + 1\, e^{-2t}\begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

$$= \begin{pmatrix} 5e^{3t} + 2e^{-2t} \\ 5e^{3t} - 3e^{-2t} \end{pmatrix},$$

since $\begin{pmatrix} 7 \\ 2 \end{pmatrix} = 5\begin{pmatrix} 1 \\ 1 \end{pmatrix} + 1\begin{pmatrix} 2 \\ -3 \end{pmatrix}$.

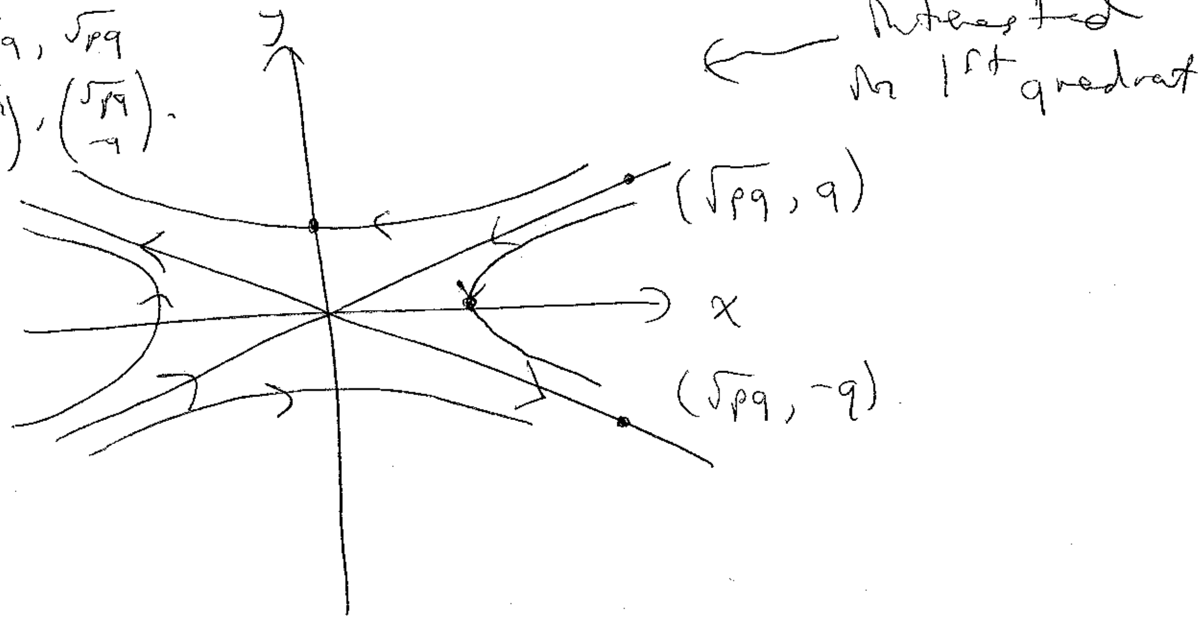
(32) $\dfrac{d\vec{x}}{dt} = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}\vec{x}$.



(46) a) The greater p or q is, the more vicious or efficient the corresponding fighter (respectively, y and x).

b) evals: $-\sqrt{pq}$, $\sqrt{pq}$

evects: $\begin{pmatrix}\sqrt{pq}\\ q\end{pmatrix}$, $\begin{pmatrix}\sqrt{pq}\\ -q\end{pmatrix}$.

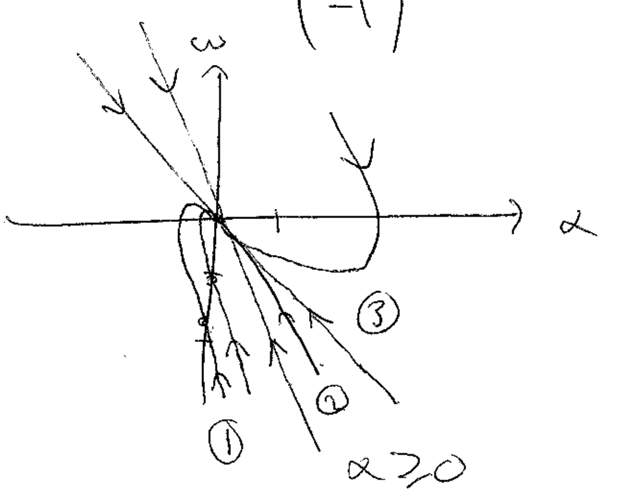


c) If $\frac{x}{y} < \sqrt{\frac{p}{q}}$, x dies out (and y wins).

If $\frac{x}{y} > \sqrt{\frac{p}{q}}$, y dies out (and x wins).

(54) a) evals: -1, -2

evects: $\begin{pmatrix}1\\ -1\end{pmatrix}$, $\begin{pmatrix}1\\ -2\end{pmatrix}$.



b) ① the door slams $\left(\frac{y}{x} < -2\right)$

② the door slows to a stop $\left(-2 < \frac{y}{x} < -1\right)$

③ the door initially swings open (if it starts early enough in the trajectory), then slows to a stop. $\left(\frac{y}{x} > -1\right)$