Last Name: _____

First Name: _____

Math 21b
First Midterm
March 7, 2000

Please circle your section:

Christine Taylor	Alex Popa	Bo Cui	Dmitry Tamarkin	John Boller	John Boller
10 MWF	11 MWF	11 MWF	12 MWF	10 TTh	11:30 TTh

Problem	Points	Score
1	9	
2	11	
3	10	
4	11	
5	8	
6	11	
Total	60	

1. (9 points)

(a) (6 points) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation with $n > m$. Show that $\ker(T) \neq \{0\}$.

(b) (3 points) Find an example of a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ with $\ker(T) = \{0\}$.

2. (11 points) The points $(2,2)$, $(-1,1)$, and $(-2,-6)$ all lie on a circle in $\mathbb{R}^2$ with equation $x^2 + y^2 + cx + dy + e = 0$.

(a) (3 points) Write the system of linear equations which will determine c,d, and e.

(b) (6 points) Write the *augmented matrix* for this system and find its *reduced row echelon form.*

(c) (2 points) Find all solutions to this system, and identify the center and radius of the circle.

3. (10 points) Let $v_1 = (1,1)$ and $v_2 = (-1,1)$ be two vectors in $\mathbb{R}^2$. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T(v_1) = (1,1)$ and $T(v_2) = (0,2)$.

(a) (5 points) Find the matrix for T.

(b) (5 points) Show that T is a *shear*.

4. (11 points) Let A be a 3×3 matrix such that $A^2 = 0$. (That is, the product of A with itself is the zero matrix.)

(a) (4 points) Show that $\mathrm{Im}(A)$ is a subspace of $\ker(A)$.

(b) (4 points) Determine all *possible* values for $\mathrm{rank}(A)$, and justify your answer(s).

(c) (3 points) Give an example of such a matrix A for each possible rank.

5. (8 points) Show that the three vectors $v_1 = (2, -3, 4)$, $v_2 = (2, -5, 2)$, and $v_3 = (-4, 5, -9)$ all lie in the same plane. What does this say about their linear independence?

6. (11 points) Let $A = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$.

(a) (5 points) Find a matrix B such that $B^2 = A$.

(b) (4 points) Determine rank(B) with justification. *(Hint: It is possible to answer part (b) without having solved part (a).)*

(c) (2 points) Determine B^{17}.