Last Name:______________________________

First Name:______________________________

Mathematics 21b

First Exam
March 5, 2001

Your Section (circle one):

Richard	Richard	Stephanie	Oliver	Daniel	Alexander
Taylor	Taylor	Yang	Knill	Allcock	Braverman
MWF 10	MWF 11	MWF 11	MWF 12	TuTh 10	TuTh 11:30

Question	Score
1	
2	
3	
4	
5	
Total	

The exam will last 90 minutes.
The exam consists of 5 questions each worth 10 points.
No calculators are allowed.
Justify your answers carefully (except in Questions 1 and 2).
For Questions 3–5, no credit can be given for unsubstantiated answers.
Write your final answers in the spaces provided.

(1) True or False (no explanation is necessary).

T F : A system of three linear equations can have exactly three solutions.

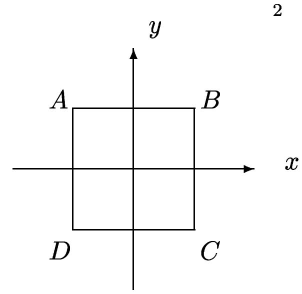
T F : If A is a square matrix such that $A^2 = 0$ then $A = 0$.

T F : There is a 2×2 matrix B with $B^2 = -I_2$.

T F : For any matrix A, the kernel of A equals the kernel of $rref\,(A)$.

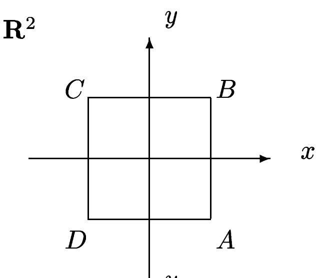
T F : The vectors $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ are linearly dependent.

(2) Consider the following square in the plane.

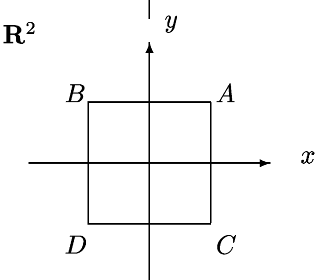


True or False (no explanation is necessary).

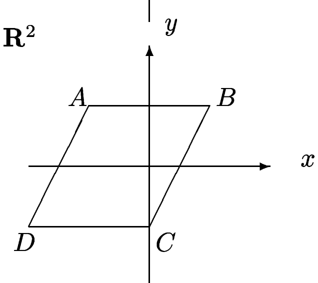
T F : There is a linear transformation $\mathbf{R}^2 \to \mathbf{R}^2$ which takes the above square to



T F : There is a linear transformation $\mathbf{R}^2 \to \mathbf{R}^2$ which takes the above square to



T F : There is a linear transformation $\mathbf{R}^2 \to \mathbf{R}^2$ which takes the above square to



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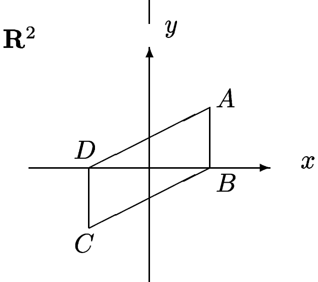

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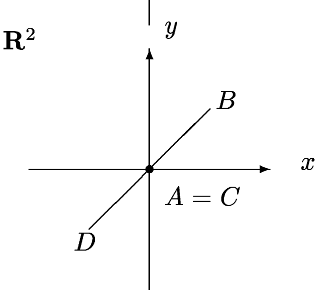

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(3) Consider the matrix

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 3 & 2 \\ 1 & -2 & 0 & 0 \\ 2 & 0 & 4 & 1 \end{bmatrix}.$$

(a) Use a series of elementary row operations to find $rref(A)$. [Do only one elementary row operation at each step.]

(b) Find a basis for the kernel of A.

(c) Find a basis for the image of A.

(d) Find a non-zero 4×4 matrix B such that $AB = 0$.

(4) (a) Consider the linear system

$$\begin{aligned} x &+ ky &= 0 \\ x &+ k^2y &= k^2. \end{aligned}$$

For which values of the constant k does this system have (i) no solutions (ii) one solution (iii) more than one solution?

(b) What is the orthogonal projection of $\begin{bmatrix} 3 \\ -4 \end{bmatrix}$ onto the line $x + 2y = 0$?

(5) Suppose that A represents a shear $T : \mathbf{R}^2 \to \mathbf{R}^2$ parallel to a line L (and that A is not the identity matrix I_2).

(a) What is the kernel of $A - I_2$?

(b) What is the image of $A - I_2$?

(c) What is $(A - I_2)^2$?