

Last Name: _____

First Name: _____

Mathematics 21b

First Midterm
March 16, 1999

Your Section (circle one):

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Liu	Cui	Polishchuk	Engelward	Winters	Winters	Dowker	Zha
MWF 9	MWF 10	MWF 11	MWF 11	MWF 12	TuTh 10	TuTh 11:30	TuTh 11:30

Question	Score
1	
2	
3	
4	
5	
Total	

No calculators are allowed.

Justify your answers carefully (except in Questions 1 and 2).

Except for question 1, no credit can be given for unsubstantiated answers.

(1) True or False (no explanation is necessary).

T F : The columns of an orthogonal matrix are always orthogonal.

T F : If A and B are symmetric $n \times n$ matrices, then AB is symmetric.

T F : The matrix of an orthogonal projection is an orthogonal matrix.

T F : If A is a 2×7 matrix and B is a 6×2 matrix, then the rank of BA is at least 5.

T F : If A is a 5×5 matrix with $A^7 = I$, then A is invertible.

T F : If A is the matrix of a reflection in a plane in R^3 , then $A^2 = I$.

T F : If B is the matrix of a rotation in R^2 such that $B^2 = I$ and $B \neq I$, then B must be $\begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix}$.

- (2) a) Let
- M
- be the matrix

$$\begin{pmatrix} 1 & 2 & 7 & 3 & 1 \\ 2 & 3 & 8 & 4 & 2 \\ 3 & 4 & 9 & 5 & 3 \\ 4 & 5 & 1 & 6 & 4 \\ 5 & 9 & 1 & 7 & 5 \end{pmatrix}$$

Is M invertible? Justify your answer briefly. (No calculation is necessary).

- b) If $A = \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}$ and $AB = \begin{pmatrix} 9 & 10 \\ 7 & 7 \end{pmatrix}$, find the matrix B .

- (3) Let
- A
- be the matrix

$$\begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$$

- a) Find the rank of A .
 b) Find the basis for $\ker(A)$.
 c) Find a basis for $\text{im}(A)$.
 d) Use Gram-Schmidt to convert the basis you found in part c) into an orthonormal basis.

- (4) Consider the linear system

$$\begin{cases} x + 2y + z = 3 \\ x + y - z = 2 \\ x + 2y + k^2z = k + 4 \end{cases}$$

- a) For which value(s) of k does this system have a unique solution?
 b) For which value(s) of k does this system have infinitely many solutions? For any such k , find all solutions.
 c) For which value(s) of k is this system inconsistent.
- (5) a) Suppose A is the matrix of a rotation about a line L in R^3 . If v is a vector in the direction of L , show that $\ker(A - 1_3)$ is spanned by v . You may assume that the angle of rotation is non-zero.
 b) Let B be the matrix for rotation through 90° clockwise about the z -axis (clockwise as seen when facing the origin from the positive z -axis). Find B and C .
 c) Find BC .
 d) It is a fact that the composition of two rotations in R^3 is another rotation. Find a vector v which spans the axis of the rotation corresponding to BC . (Hint: Use part a).