True or False

1. T; The matrix is $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$.

2. F; The columns of a rotation matrix are unit vectors; see Fact 2.2.2.

3. T, by Fact 2.3.3.

4. T; Let $A = B$ in Fact 2.4.8.

5. F, by Fact 2.4.3.

6. T, by Fact 2.4.9.

7. F; Matrix AB will be 3×5, by Definition 2.4.1b.

8. F; Note that $T \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. A linear transformation transforms $\vec{0}$ into $\vec{0}$.

9. T, by Fact 2.2.3.

10. T, by Fact 2.4.5.

11. F, by Fact 2.3.6. Note that the determinant is 0.

12. T, by Fact 2.3.3.

13. T; The shear matrix $A = \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 1 \end{bmatrix}$ works.

14. T; Simplify to see that $T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4y \\ -12x \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ -12 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$.

15. T; The equation $\det(A) = k^2 - 6k + 10 = 0$ has no real solution.

16. T; The matrix fails to be invertible for $k = 5$ and $k = -1$, since the determinant is 0 for these values.

17. F; Note that $\det(A) = (k-2)^2 + 9$ is always positive, so that A is invertible for all values of k.

18. T; Note that the columns are unit vectors, since $(-0.6)^2 + (\pm 0.8)^2 = 1$. The matrix has the form presented in Fact 2.2.2.

19. F; Consider $A = I_2$ (or any other invertible 2×2 matrix).

20. T; Note that $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}^{-1}$ is the unique solution.

21. F; For any 2×2 matrix A, the two columns of $A \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ will be identical.

22. T; One solution is $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$.

23. T; Note that $\begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, so that it's a shear parallel to the line spanned by vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

24. T; Just multiply it out.

25. T; The product is $\det(A) I_2$.

26. T; Writing an upper triangular matrix $A = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ and solving the equation $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ we find that $A = \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$, where b is any nonzero constant.

27. T; Note that the matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ represents a rotation through $\pi/2$. Thus $n = 4$ (or any multiple of 4) works.

28. F; If a matrix A is invertible, then so is A^{-1}. But $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ fails to be invertible.

29. F; If matrix A has two identical rows, then so does AB, for any matrix B. Thus AB cannot be I_n, so that A fails to be invertible.

30. T, by Fact 2.3.4. Note that $A^{-1} = A$ in this case.

31. F; Consider the matrix A that represents a rotation through the angle $2\pi/17$.

32. F; Consider the reflection matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

33. T; We have $(5A)^{-1} = \frac{1}{5} A^{-1}$.

34. T; The equation $A\vec{e}_i = B\vec{e}_i$ means that the ith columns of A and B are identical. This observation applies to all the columns.

35. T; Note that $A^2B = AAB = ABA = BAA = BA^2$.

36. T; Multiply both sides of the equation $A^2 = A$ with A^{-1}.

37. F; Consider $A = I_2$ and $B = -I_2$.

38. T; Since $A\vec{x}$ is on the line onto which we project, the vector $A\vec{x}$ remains unchanged when we project again: $A(A\vec{x}) = A\vec{x}$, or $A^2\vec{x} = A\vec{x}$, for all $\vec{x}$. Thus $A^2 = A$.

39. F; Consider matrix $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, for example.

40. T; Apply Fact 2.4.9 to the equation $(A^2)^{-1}AA = I_n$, with $B = (A^2)^{-1}A$.

41. T; If you reflect twice in a row (across the same line), you will get the original vector back: $A(A\vec{x}) = \vec{x}$, or, $A^2\vec{x} = \vec{x} = I_2\vec{x}$. Thus $A^2 = I_2$ and $A^{-1} = A$.

42. F; Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, for example.

43. T; Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$, for example.

44. F; By Fact 1.3.3, there is a nonzero vector $\vec{x}$ such that $B\vec{x} = \vec{0}$, so that $AB\vec{x} = \vec{0}$ as well. But $I_3\vec{x} = \vec{x} \neq \vec{0}$, so that $AB \neq I_3$.

45. T; We can rewrite the given equation as $A^2 + 3A = -4I_3$ and $-\frac{1}{4}(A + 3I_3)A = I_3$. By Fact 2.4.9, matrix A is invertible, with $A^{-1} = -\frac{1}{4}(A + 3I_3)$.

46. T; Note that $(I_n + A)(I_n - A) = I_n^2 - A^2 = I_n$, so that $(I_n + A)^{-1} = I_n - A$.

47. T; If we shear parallel to line L, then the vector $A\vec{x} - \vec{x}$ is parallel to L, for all $\vec{x}$, by part b of Definition 2.2.4. By part a, we have $A(A\vec{x} - \vec{x}) = A\vec{x} - \vec{x}$, or $(A^2 - 2A + I_2)\vec{x} = \vec{0}$. Since this holds for all vectors $\vec{x}$ in $\mathbb{R}^2$, the equation $A^2 - 2A + I_2 = 0$ must hold.

48. F; Consider $T(\vec{x}) = 2\vec{x}$, $\vec{v} = \vec{e}_1$, and $\vec{w} = \vec{e}_2$.

49. F; Since there are only eight entries that are not 1, there will be at least two rows that contain only ones. Having two identical rows, the matrix will be non-invertible.

50. F; Let $A = B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, for example.