27. Use Fact 9.1.2.

The eigenvalues of $A = \begin{bmatrix} -4 & 3 \\ 2 & -3 \end{bmatrix}$ are $\lambda_1 = -6$ and $\lambda_2 = -1$, with associated eigenvectors $\vec{v}_1 = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. The coordinates of $\vec{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ with respect to $\vec{v}_1$ and $\vec{v}_2$ are $c_1 = -\frac{1}{5}$ and $c_2 = \frac{2}{5}$.

By Fact 9.1.2 the solution is $\vec{x}(t) = -\frac{1}{5}e^{-6t}\begin{bmatrix} -3 \\ 2 \end{bmatrix} + \frac{2}{5}e^{-t}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

28. $\lambda_1 = 2$, $\lambda_2 = 10$; $\vec{v}_1 = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$; $c_1 = -\frac{1}{8}$, $c_2 = \frac{5}{8}$, so that $\vec{x}(t) = -\frac{1}{8}e^{2t}\begin{bmatrix} -3 \\ 2 \end{bmatrix} + \frac{5}{8}e^{10t}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

40. $\vec{x}(t) = e^{2t}\begin{bmatrix} 2 \\ 3 \end{bmatrix} + e^{3t}\begin{bmatrix} 3 \\ 4 \end{bmatrix}$

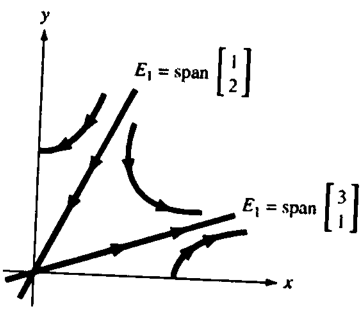
We want a 2×2 matrix A with eigenvalues $\lambda_1 = 2$ and $\lambda_2 = 3$ and associated eigenvectors $\vec{v}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$; that is $A\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 9 \\ 6 & 12 \end{bmatrix}$ or $A = \begin{bmatrix} 4 & 9 \\ 6 & 12 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} 4 & 9 \\ 6 & 12 \end{bmatrix}\begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}$

$= \begin{bmatrix} 11 & -6 \\ 12 & -6 \end{bmatrix}$.

/ / /

42. a. The term $0.8x$ in the second equation indicates that species y is helped by x, while species x is hindered by y (consider the term $-1.2y$ in the first equation). Thus y preys on x.

b.



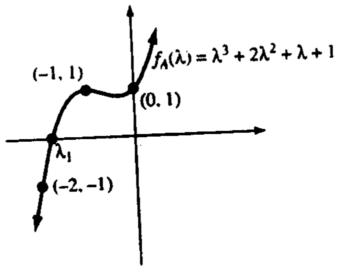
c. If $\frac{y(0)}{x(0)} < 2$ then both species will prosper, and $\lim_{t\to\infty} \frac{y(t)}{x(t)} = \frac{1}{3}$.

If $\frac{y(0)}{x(0)} \geq 2$ then both species will die out.

43. a. These two species are *competing* as each is hindered

9. The eigenvalues are conjugate complex, $\lambda_{1,2} = p \pm iq$, and $\text{tr}(A) = 2p < 0$, so that p is negative. By Fact 9.2.4, the zero state is stable.

12. Yes, the zero state is stable. To see this, use technology to determine that the real parts of the eigenvalues are all negative. Or draw a rough sketch of the characteristic polynomial to see that there is one real eigenvalue λ_1 between -1 and -2; the two other eigenvalues must be $\lambda_{2,3} = p \pm iq$ with $p < 0$, since $\lambda_1 + \lambda_2 + \lambda_3 = \lambda_1 + 2p = \text{tr}(A) = -2$.



...ll that the zero state is stable if (and only if) the real parts of all eigenvalues are negative. ... real parts have the same sign

16. If $A = \begin{bmatrix} 0 & 1 \\ a & b \end{bmatrix}$ then $\text{tr}(A) = b$ and $\det(A) = -a$. By Fact 9.2.5, the zero state is stable if a and b are both negative.

17. If $A = \begin{bmatrix} -1 & k \\ k & -1 \end{bmatrix}$ then $\text{tr}(A) = -2$ and $\det(A) = 1 - k^2$. By Fact 9.2.5, the zero state is stable if $\det(A) = 1 - k^2 > 0$, that is, if $|k| < 1$.

18. If $\lambda_1, \lambda_2, \lambda_3$ are real and negative, then $\text{tr}(A) = \lambda_1 + \lambda_2 + \lambda_3 < 0$ and $\det(A) = \lambda_1\lambda_2\lambda_3 < 0$. If λ_1 is real and negative and $\lambda_{2,3} = p \pm iq$, where p is negative, then $\text{tr}(A) = \lambda_1 + 2p < 0$ and $\det(A) = \lambda_1(p^2 + q^2) < 0$. Either way, both trace and determinant are negative.

26. $\lambda_1 = 1, \lambda_2 = -2; E_1 = \text{span}\begin{bmatrix} 0 \\ 1 \end{bmatrix}, E_{-2} = \text{span}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

System is continuous so choose V.

8. $p_T(\lambda) = \lambda^2 + 3\lambda - 10 = (\lambda + 5)(\lambda - 2) = 0$
$x(t) = c_1 e^{-5t} + c_2 e^{2t}$, where c_1, c_2 are arbitrary constants.

9. $p_T(\lambda) = \lambda^2 - 9 = (\lambda - 3)(\lambda + 3) = 0$
$f(t) = c_1 e^{3t} + c_2 e^{-3t}$, where c_1, c_2 are arbitrary constants.

10. $p_T(\lambda) = \lambda^2 + 1 = 0$ has roots $\lambda_{1,2} = \pm i$. By Fact 9.3.9, $f(t) = c_1 \cos(t) + c_2 \sin(t)$, where c_1, c_2 are arbitrary constants.

11. $p_T(\lambda) = \lambda^2 - 2\lambda + 2 = 0$ has roots $\lambda_{1,2} = 1 \pm i$. By Fact 9.3.9, $x(t) = e^t(c_1 \cos(t) + c_2 \sin(t))$, where c_1, c_2 are arbitrary constants.

12. $p_T(\lambda) = \lambda^2 - 4\lambda + 13 = 0$ has roots $\lambda_{1,2} = 2 \pm 3i$. By Fact 9.3.9, $f(t) = e^{2t}(c_1 \cos(3t) + c_2 \sin(3t))$, where c_1, c_2 are arbitrary constants.

27. General solution $f(t) = c_1 \cos(3t) + c_2 \sin(3t)$ (Fact 9.3.9)
Plug in: $0 = f(0) = c_1$ and $1 = f\left(\frac{\pi}{2}\right) = -c_2$, so that $c_1 = 0, c_2 = -1$, and $f(t) = -\sin(3t)$.

28. General solution $f(t) = c_1 e^{-4t} + c_2 e^{3t}$, with $f'(t) = -4c_1 e^{-4t} + 3c_2 e^{3t}$
Plug in: $0 = f(0) = c_1 + c_2$ and $0 = f'(0) = -4c_1 + 3c_2$, so that $c_1 = c_2 = 0$ and $f(t) = 0$.