Name: ____

Math 21b Midterm 1 Thursday, October 24th, 2002

Please circle your section:

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Katherine Visnjic (CA)	Jakub Topp (CA)	Erin Aylward (CA)
MWF 9-10	MWF 10-11	MWF 11-12

Question	Points	Score
1	18	
2	15	
3	18	
4	12	
5	15	
6	12	
7	10	
Total	100	

You have two hours to take this midterm. Pace yourself by keeping track of how many problems you have left to go and how much time remains. You don't have to answer the problems in any particular order. So move on to another problem if you find you're stuck and that you are spending too much time on one problem.

To receive full credit on a problem, you will need to justify your answers carefully - unsubstantiated answers, even if correct, will receive little or no credit (except if the directions for that question specifically say no justification is necessary, such as the True/False).

Please be sure to write neatly - illegible answers will also receive little or no credit.

If more space is needed, use the back of the previous page to continue your work. Be sure to make a note of that so that the grader knows where to find your answers.

You are allowed one 3 by 5 inch file card with formulas on it during the test, but you are not allowed to use any other notes, or calculators during this test.

Good luck! Focus and do well!

Question 1. (18 points total)

True or False (3 points each) No justification is necessary, simply circle **T** or **F** for each statement.

T **F** (a) If A is invertible then the number of rows of A must equal the rank of A.

T **F** (b) If A has a nonzero kernel, then A is not invertible.

T **F** (c) The only 2 x 2 matrix A with $A^2 = I_2$ is the identity matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

T **F** (d) Let A and B be two n x n matrices such that $\ker(B) \neq \ker(A)$. Then the matrix AB is not invertible.

T **F** (e) If A is a 5 x 4 matrix of rank 4 then the kernel of A must be nonzero.

T **F** (f) Let A and B be two matrices such that the product AB is defined. If B has a column of zeros, then AB must also have a column of zeros.

Question 2 (15 points total)

Suppose that $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ are matrices for two linear transformations.

(a) (4 points) Briefly describe the two transformations given by *A* and *B* geometrically.

(b) (4 points) What are the ranks of *A* and *B*? Is either *A* or *B* invertible? If not, justify your answer, if so then find its inverse.

(c) (4 points) Do *A* and *B* commute? Justify your answer by describing the transformations AB and BA geometrically.

(d) (3 points) What is A^{2002}?

Question 3. (18 points total)

(a) (10 points) Find all solutions to the following homogeneous linear system (if there is more than one solution, express the solutions in vector form, i.e. as a linear combination of column vectors).

$$\left\{ \begin{array}{rrrrl} 2x_1 & - x_2 & + 3x_3 & + 3x_4 & = 0 \\ -2x_1 & & + x_3 & + 4x_4 & = 0 \\ 10x_1 & - 3x_2 & + 7x_3 & + x_4 & = 0 \end{array} \right\}$$

(b) (3 points) Is it possible to give a nontrivial solution to the linear system in part (a) with $x_3 = x_4 = 0$? If so, write down one such solution, if not justify your answer

Question 3 continued

(c) (5 points) Find all values for λ for which the following homogeneous linear system has nontrivial solutions.

$$\begin{cases} 2x_1 \qquad + x_3 = 0 \\ \quad -\lambda x_2 + x_3 = 0 \\ \quad (1-\lambda^2)x_3 = 0 \end{cases}$$

Question 4. (12 points total)

Suppose you know that the inverses of two 3 x 3 matrices, *A* and *B*, are given as follows:

$$A^{-1} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -3 & 1 \\ 2 & -2 & 0 \end{bmatrix} \text{ and } B^{-1} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

(a) (4 points) Find all solutions to $B\vec{x} = \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}$

(b) (4 points) Find all solutions to $A\vec{x} = \vec{0}$

(c) (4 points) Consider the transformation $T(\vec{x}) = (BA)\vec{x}$, where A and B are the same two matrices as in parts (a) and (b). Find the matrix for the inverse transformation, $T^{-1}(\vec{x})$

Question 5. (15 points total)

Find examples of linear transformations that have the following features (i.e. write down a possible matrix for each of the transformations):

(a) (3 points) Find an example of a transformation $T:\mathbb{R}^3 \to \mathbb{R}^4$ that has a nonzero kernel.

(b) (3 points) Find an example of a transformation $T:\mathbb{R}^3 \to \mathbb{R}^4$ that has image equal to the line spanned by $\vec{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$

(c) (3 points) Find an example of a transformation $T:\mathbb{R}^3 \to \mathbb{R}^4$ that has a kernel equal to its domain.

(d) (3 points) Is it possible to find an example of a transformation $T:\mathbb{R}^3 \to \mathbb{R}^4$ that has an image equal to its codomain? If so, give an example, if not, explain why not.

(e) (3 points) Is it possible to find an example of a transformation $T:\mathbb{R}^3 \to \mathbb{R}^4$ so that $T^2 = I_3$. If so, give an example, if not, explain why not.

Question 6. (12 points total)

(a) (8 points) Find all values of □ for which the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 2 & □ \end{bmatrix}$ exists and write down the matrix for A^{-1}.

(b) (4 points) Suppose □ is chosen to be equal to 1 (so that the matrix A^{-1} does in fact exist) then calculate the matrix product $\left(A^{-1} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} A \right)^2$

Question 7. (10 points total)

Let A be an n x n invertible matrix and let B be an n x r matrix. Show that $\ker(AB) = \ker(B)$
Hint: First show that $\ker(B) \subseteq \ker(AB)$ *by taking a vector in* $\ker(B)$ *and show that this vector must also be in* $\ker(AB)$. *Then show that* $\ker(AB) \subseteq \ker(B)$