Last Name:________________________

First Name:________________________

Mathematics 21b

First Midterm Examination
October 25, 1999

Your Section (circle one):

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MWF 10	MWF 11

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
Total	40	

No calculators are allowed.

1. Consider the system of linear equations

$$\begin{cases} 3x_3 - 2x_2 = -3, \\ x_3 + 2x_4 + 4x_1 = 1, \\ 2x_1 + x_2 - x_3 + x_4 = 2. \end{cases}$$

(a) (2 pts.) Write the coefficient matrix and augmented matrix for this system.

(b) (5 pts.) Calculate the row-reduced echelon form of the augmented matrix.

(c) (3 pts.) Find the general solution of the linear system. Verify that your answer does in fact satisfy all the equations.

2. Let A and B be the matrices

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}.$$

(a) (4 pts.) Describe A, B geometrically as linear transformations.

(b) (3 pts.) What are the ranks of A and B? Is either A or B invertible? Justify your answers.

(c) (3 pts.) Do A and B commute? Interpret your result geometrically.

3. (a) (1 pt. each) Define:

- kernel

- image

- rank

- span

- basis

(b) If A is a 4×5 matrix of rank 4,

- (1 pt.) Is A invertible? Why?

- (1 pt.) What can you say about $\ker(A)$?

- (1 pt.) What can you say about $\text{im}(A)$?

- (2 pts.) What can you say about a linear system whose coefficient matrix is A?

4. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be orthogonal projection to the subspace of $\mathbb{R}^3$ with basis consisting of the single vector $\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$. [Note: this is not a unit vector!]

(a) (4 pts.) Calculate $T\vec{e}_1$, $T\vec{e}_2$, $T\vec{e}_3$.

(b) (2 pts.) Find the matrix for T.

(c) (4 pts.) What are the dimensions of $\ker(T)$ and $\operatorname{im}(T)$? Find bases for these subspaces.

You must justify your answers to receive full credit.