Name: _____

Math 21b Midterm 2 Tuesday, November 19th, 2002

Please circle your section:

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MWF 9-10	MWF 10-11	MWF 11-12

Question	Points	Score
1	18	
2	14	
3	12	
4	16	
5	16	
6	10	
7	14	
Total	100	

You have two hours to take this midterm. Pace yourself by keeping track of how many problems you have left to go and how much time remains. You don't have to answer the problems in any particular order. So move on to another problem if you find you're stuck and that you are spending too much time on one problem.

To receive full credit on a problem, you will need to justify your answers carefully - unsubstantiated answers, even if correct, will receive little or no credit (except if the directions for that question specifically say no justification is necessary, such as the True/False).

Please be sure to write neatly - illegible answers will also receive little or no credit.

If more space is needed, use the back of the previous page to continue your work. Be sure to make a note of that so that the grader knows where to find your answers.

You are allowed a half page of notes on it during the test, but you are not allowed to use any other references or calculators during this test.

Good luck! Focus and do well!

Question 1. (18 points total)

True or False (3 points each) No justification is necessary, simply circle **T** or **F** for each statement.

T **F** (a) If A is an invertible matrix then the kernels of A and A^{-1} must be equal.

T **F** (b) If A is a 5×8 matrix then it is possible for the dimension of kernel(A) to equal two.

T **F** (c) If a subspace V of $\Re^3$ contains two linearly independent vectors then V must contain at least one of the standard basis vectors as well.

T **F** (d) If the product AB of two $n \times n$ matrices is the zero matrix, then BA must also equal the zero matrix.

T **F** (e) If A is a (square) orthogonal matrix, and the product AB is also orthogonal, then the matrix B must be orthogonal as well.

T **F** (f) If $A^T A = AA^T$ for an $n \times n$ matrix A, then A must be an orthogonal matrix.

Question 2 (14 points total)

Suppose that $A = \begin{bmatrix} 1 & 2 & 3 & -1 & 4 \\ 0 & 3 & 3 & -3 & 6 \\ 1 & 1 & 2 & 0 & 2 \\ 2 & 0 & 2 & 2 & 0 \end{bmatrix}$. Find a basis for each of the following subspaces.

(a) (6 points) Image(A).

(b) (6 points) Kernel(A)

(c) (2 points) What is the dimension of the kernel (A^{T})?

Question 3. (12 points total)

(a) (6 points) Let $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m$ be a set of vectors that span a subspace V. Suppose $\vec{w}$ is another vector in the subspace V. Show that the set of vectors $\vec{w}, \vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m$ is linearly dependent.

(b) (6 points) Suppose that $\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_k$ is a set of k linearly independent vectors in $\Re^n$. Suppose A is an $n \times n$ invertible matrix, and that $\vec{y}_i = A\vec{x}_i$ for i = 1 to k (i.e. $\vec{y}_1 = A\vec{x}_1, \vec{y}_2 = A\vec{x}_2, \ldots, \vec{y}_k = A\vec{x}_k$). Show that set of vectors $\vec{y}_1, \vec{y}_2, \ldots, \vec{y}_k$ is also linearly independent.

Question 4. (16 points total)

(a) (10 points) Find an orthonormal basis for the kernel of the matrix $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}$

(b) (6 points) Suppose that $A = BC$ where B is a 4 by 3 matrix and C is a 3 by 4 matrix. Is it possible for A to be an invertible matrix? If so, give an example (write down matrices A, B and C). If not, explain why not.

Question 5. (16 points total)

(a) (6 points) Write down a basis for the linear space of all skew-symmetric 3×3 matrices (recall that A is skew-symmetric if $A^T = -A$), and thus determine the dimension of this space.

(b) (6 points) Find the dimension of the linear space of all symmetric $n \times n$ matrices (recall that A is symmetric if $A^T = A$)

(c) (4 points) If A is an $n \times n$ symmetric matrix, then is A^2 necessarily symmetric as well? Explain why or why not.

Question 6. (10 points total)

Use the method of least squares to find the linear function $y = mx + b$ that best fits the following data:

x	-6	-2	1	7
y	-1	2	1	6

Question 7. (14 points total)

(a) (10 points) Let T: $V \rightarrow V$ be a linear transformation, where V is a linear space. Suppose that kernel(T^2) = kernel(T). Show that it must also be the case that kernel(T^3) = kernel(T^2).

As before, a good strategy is to first show that kernel(T^2) $\subseteq$ kernel(T^3), and then show that kernel(T^3) $\subseteq$ kernel(T^2). (Note, V is a general linear space, so don't assume that V is equal to $\Re^n$ in your answer).

(b) (4 points) Give an example of a nonzero linear transformation T from $\Re^3$ to $\Re^3$ such that kernel(T^2) is equal to kernel(T).