True or False

1. The image of a 3×4 matrix is a subspace of $\mathbb{R}^4$.

2. The column vectors of a 5×4 matrix must be linearly dependent.

3. If a subspace V of $\mathbb{R}^n$ contains none of the standard vectors $\mathbf{e}_1, \ldots, \mathbf{e}_n$, the V consists of the zero vector only.

4. If the image of an $n \times n$ matrix A is all of $\mathbb{R}^n$, then A must be invertible.

5. If A and B are $n \times n$ matrices and $\mathbf{v}$ is in the kernel of both A and B, then $\mathbf{v}$ is also in the kernel of AB.

6. The vectors
$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$
form a basis for $\mathbb{R}^3$.

7. If $2\mathbf{u} + 3\mathbf{v} + 4\mathbf{w} = 5\mathbf{u} + 6\mathbf{v} + 7\mathbf{w}$, then the vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ must be linearly dependent.