# Invertible Matrix Theorem

Let $A$ be an $n \times n$ matrix. The following statements are equivalent.

1. $A$ is invertible.
2. $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.
3. $A\mathbf{x} = \mathbf{b}$ has a unique solution.
4. rref$(A) = I_n$.
5. rank$(A) = n$
6. im$(A) = \mathbb{R}^n$
7. ker$(A) = \{\mathbf{0}\}$
8. The columns of $A$ are linearly independent.
9. The columns of $A$ form a basis for $\mathbb{R}^n$.
10. The columns of $A$ span $\mathbb{R}^n$