Invertible Matrix Theorem

Let A be an $n \times n$ matrix. The following statements are equivalent.

1. A is invertible.

2. $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.

3. $A\mathbf{x} = \mathbf{b}$ has a unique solution.

4. $\text{rref}(A) = I_n$.

5. $\text{rank}(A) = n$

6. $\text{im}(A) = \mathbb{R}^n$

7. $\text{ker}(A) = \{\mathbf{0}\}$

8. The columns of A are linearly independent.

9. The columns of A form a basis for $\mathbb{R}^n$.

10. The columns of A span $\mathbb{R}^n$

11. $\det(A) \neq 0$

12. 0 is not an eigenvalue of A