

## Invertible Matrix Theorem

Let  $A$  be an  $n \times n$  matrix. The following statements are equivalent.

1.  $A$  is invertible.
2.  $A\mathbf{x} = \mathbf{0}$  has only the trivial solution.
3.  $A\mathbf{x} = \mathbf{b}$  has a unique solution.
4.  $\text{rref}(A) = I_n$ .
5.  $\text{rank}(A) = n$
6.  $\text{im}(A) = \mathbb{R}^n$
7.  $\text{ker}(A) = \{\mathbf{0}\}$
8. The columns of  $A$  are linearly independent.
9. The columns of  $A$  form a basis for  $\mathbb{R}^n$ .
10. The columns of  $A$  span  $\mathbb{R}^n$