

Math 21b

Linear Algebra and Differential Equations

Spring 2003

Syllabus

Course Head:

Thomas W. Judson
Office: SC 435
Tel: (617) 495-4744
Email: judson@math.harvard.edu

Orientation Meeting:

Wednesday, January 29, 2003 in Science Center D at 8:30 am.

Sectioning:

You must section by noon on Thursday, January 30. If you have an email account, log on to the Harvard computer system, then type

ssh section@ulam.fas.harvard.edu

instead of "pine," and follow the instructions.

Catalogue Description:

By working with vectors and matrices, linear algebra provides the structure for solving problems that arise in practical applications ranging from Markov processes to optimization and from Fourier series to statistics. To understand how, we develop thorough treatments of: Euclidean spaces, including their bases, dimensions and geometry; and linear transformation of such spaces, including their determinants, eigenvalues, and eigenvectors. These concepts will be applied to solve dynamical systems, including both ordinary and partial differential equations.

Note:

Required first meeting in spring: Wednesday, January 29, 8:30 am in Science Center D. May not be taken for credit by students who have passed Applied Mathematics 21b. This course, when taken for a letter grade, meets the Core area requirement for Quantitative Reasoning.

Prerequisite: Mathematics 1b or equivalent. Mathematics 21a is commonly taken before Mathematics 21b, but is not a prerequisite, although familiarity with partial derivatives is useful.

About Linear Algebra:

Up until now your math classes have probably concentrated on helping you master calculus. Along the way you have seen such things as triple integrals and vector fields, and it is likely that you have studied some differential equations as well. Life after calculus is linear algebra.

Our world is multidimensional, and linear algebra provides us with tools to help to handle this through matrix manipulation, the study of dynamical systems, and an understanding of eigenvalues and eigenvectors. After taking Math 21b, you will have finished your study of three of the most important mathematics topics: calculus, differential equations, and linear algebra. What you will learn about linear algebra this semester will be useful in many different fields, from economics to epidemiology, and from physics to the social sciences, and of course, it will be extremely useful if you go on in mathematics as well.

In calculus you worked with the ideas of derivatives and integration. When you studied multivariable calculus, you were introduced to the notions of vectors and multidimensional spaces. The purpose of this course is to develop your understanding and ability to deal with linearity. A nonlinear problem can often be approximated by a linear problem. In fact, the whole idea of differential calculus is to approximate a curve with a linear equation. Linear algebra also serves as a bridge to higher mathematics, since a certain emphasis is placed on the axiomatic development of the subject through the use of theorem and proof.

One of the most important problems and applications of mathematics is that of solving a system of equations. Over three-fourths of all mathematical problems encountered in business, science, engineering, economics, or other industrial applications involve solving such a system. Matrices can be used to represent systems of linear equations. At this point you might picture a matrix as simply an organized set of numbers, with a certain number of rows and columns, but they can also be thought of as representing something called a linear transformation. Linear transformations map one multidimensional space into another such space. These transformations can often be classified geometrically as rotations, reflections, dilations, etc. This geometric interpretation leads us to the idea of dynamical systems, which involves studying repeated transformations. Eigenvalues and eigenvectors are an indispensable tool, if you want to understand and solve dynamical systems.

Course Goals and Learning Objectives:

The goals for the course are to

- Appreciate the role that linearity plays in mathematics.
- Explore and understand the two central problems of linear algebra, solving the matrix equations $A\mathbf{x} = \mathbf{b}$ and $A\mathbf{x} = \lambda\mathbf{x}$.
- Learn the fundamentals of linear algebra in preparation for further applications in the physical, biological, information, or social sciences.
- Investigate the axiomatic structure of linear algebra.

Upon successfully completing this course you should be able to

- Express systems of equations in terms of matrices and be able to efficiently find and describe the solutions of such systems.
- Articulate the definition and properties of the determinant and their evaluation.
- Articulate the definition of a linear space, linear independence, basis and dimension, and linear transformations, especially in the case of \mathbb{R}^n .
- Understand the relationship between geometry and linear algebra, including the roles of inner products and orthogonality.
- Apply linear algebra to solve various problems in mathematics such as finding the least

- squares solution to a linear system $A\mathbf{x} = \mathbf{b}$.
- Articulate the eigenvalue problem and apply eigenvalues and eigenvectors to various areas of mathematics, including dynamical systems and differential equations.

Textbook:

Otto Bretscher. *Linear Algebra with Applications*, second edition. Prentice-Hall, Upper Saddle River NJ, 2001. This textbook is available at the Harvard Coop. Since the earlier first edition has a number of sections in a different order from the second edition, make sure that you get the second edition for this semester's class.

Grading:

Your course grade will be determined as follows:

- Homework 25%
- Two Midterms 20% each
- Final Exam 35%

We do not set absolute point value levels ahead of time (i.e., 90 and above equals A). The reason for this is to take into account the fact that the course and the tests vary somewhat from semester to semester, and it would be unfair to penalize the class if it turned out that scores on a particular test were lower one semester due to the nature of the test. We will indicate after each test a rough range of grade equivalence, so that you can keep track of how you are doing in the course.

Exams:

There will two exams during the semester as well as a final exam. Because of the need to have everyone take tests at a common time, something that is practically impossible to do early during the day, the midterms are both scheduled in the evening. It is your responsibility to let your section leader know as soon as possible of any potential conflicts. It is also generally the case that it is your responsibility to resolve any scheduling conflicts. There are only two of these evening tests during the semester, and they should take precedence over any other obligations that you might have. Make-up exams will be administered only if a documented serious illness or personal tragedy prevents a person from taking an exam at the scheduled time.

- First Test: Thursday, March 6, 2003 from 7:00 to 9:00 pm in Science Center Auditorium D.
- Second Test: Thursday, April 10, 2003 from 7:00 to 9:00 pm in Science Center Auditorium D.
- Final Exam: Tentatively scheduled for Tuesday, May 20, 2003 with the time a place to be announced.

Exam questions will be similar to the homework problems and examples discussed in class, but only up to a point. Be prepared to spend some time thinking during tests, not just spending time busily write down formulas.

Homework:

There is no question that the best way to learn math is by doing math, and homework exercises are an essential part of any math course. If you just go to a math class and watch the teacher

work problems, but don't actually try doing any problems on your own, then there is very little chance you will really learn the subject. It is also very unlikely that you will do well on math exams without working through homework problems ahead of time. While doing homework, don't just write down answers. Think about the problems posed, your strategies, the meaning of your computations, and the answers you get. The main point is not to come up with specific answers to the specific problems you are working on, but to develop an understanding of what you are doing so that you can apply your reasoning to a wide range of similar situations. It is very unlikely that later on in life you will see exactly the same math problems you're working on now, so learn the material in such a way that you are prepared to use your general knowledge of mathematics in the future, not just how to apply particular formulas for very specific problems.

You are encouraged to form study groups with other students in the class so that you can discuss your work with each other; however, all work submitted must be written up individually. Make sure that even if you do work in groups, that you come away with the ability to explain everything you end up writing up in your homework.

There will generally be two problem sets each week. Assignments will be graded by your course assistant and will typically be returned to you at the following class meeting. We will then post solutions to the homework on the course website. Check the solutions so that you can learn from your work. In order for us to post solutions as soon as possible, and in light of the fact that getting behind in a math class is one of the most uncomfortable things you can do to yourself, homework must be turned in on time. Since we will drop your 3 lowest homework grades, please do not try to harass your course assistant into accepting a late homework assignment. The homework policy is a course wide policy, and it would be unfair if certain course assistants were more lenient than others.

There will be times when problems for homework will look different from what is discussed in class. For some classes we might ask you to read through a section ahead of time so that when you then see it covered in class, you will be able to follow along much more easily (as opposed to seeing it for the very first time in class). As an incentive to do this pre-reading, we might ask you to do one or two very straightforward questions from that section for homework, even though the material has not been covered yet in class.

Certain assigned problems will be marked with a (*). These problems are extra credit. They are usually a little more difficult or longer, but they are also more interesting. We certainly encourage you to try as many of these problems as possible. If you are able to do them correctly, it will help your overall homework score; however, the best grade that you can receive for your homework is still only 100%.

We are also evaluating some new software for helping students learn linear algebra. Math 21b will be a pilot site for Just Ask, an online data bank of linear algebra problems (<http://www.justask4u.com/>). You will receive an account and a password later in the semester. We will have one or two problem sets over the semester where you will have to complete by connecting to the web site; we will primarily use Just Ask for exam review. We hope that this will be a helpful tool for you during the course.

Classes, Problem Sessions and Course Assistants:

Math 21b is taught in sections that meet three hours per week. The philosophy behind the sections is that it is far better to work on math in smaller groups than in one large, impersonal lecture setting. This gives you a better opportunity to ask questions in class and interact with

your teacher. Make sure you take advantage of this arrangement and try to get the most out of being in these smaller groups. Any questions you ask in class will likely be ones that other students will want answered as well, so get over any hesitation you might have and ask questions as the material is presented. There is no class participation grade, so go ahead and ask away. You will not be penalized for doing this, no matter how trivial or simple you think your questions might seem. Remember, the class is being held for you to learn the material, not just to give you a time to copy notes off of a blackboard, so be sure to get help when you need it and stay involved in your class.

You will also be attending a problem session led once per week by a Course Assistant (CA). Course Assistants grade homework and hold weekly problem sessions. They also attend the 21b classes with you, so you will get to know them well during the semester. The problem sessions are an important part of the course and will be devoted mainly to working problems and reviewing material. Even if you find you are not having difficulty doing the homework problems, you should still make a habit of attending these sessions. A schedule of all of the problem sessions will be posted in the Science Center and on our course web page, so that if you have a scheduling conflict with your particular section's problem session, then you should still be able to attend another problem session.

Math Question Center:

In addition to class, problem sessions and office hours, the math department runs a Math Question Center (MQC) in Loker on Sunday through Thursday evenings from 8:00 pm to 10:00 pm. The MQC is staffed by course assistants as well as by graduate students and other teaching staff. This is a good place to meet with other students in your class to do homework. You should feel free to drop in any time you want a bit of help, or if you just want to solidify your basic math understanding by doing some review problems.

Technology:

In general, technology is a good thing, but as with everything, sometimes too much of a good thing can lead to problems. With the advent of graphing calculators and mathematical software programs, such as Matlab and Mathematica it is now possible to do an amazing number of things almost instantaneously that would otherwise take hours or days to do by hand. For instance, try multiplying two 12 by 12 matrices together by hand. Computers can help you with your math skills and instincts by reducing the time you spend doing burdensome computations, however you should not rely on computers and calculators to such an extent that they keep you from developing your own skills. Technology should be used as an aid, but without a good understanding of the underlying mathematical concepts, the computer will quite happily mislead you without your even knowing it.

As a general policy, we encourage you to use graphing calculators or computer software during the semester as long as they are used as tools to help you learn and explore math, and not as crutches that keep you from developing your own understanding. If you wish to purchase a calculator, we recommend a Texas Instruments TI-86 calculator. Harvard University also has a site license agreement with Mathematica. You can obtain a copy for your own computer by visiting the web site: <http://www.courses.fas.harvard.edu/software/mathematica/>.

To the extent that the main point of the course is for you to develop confidence in your mathematical abilities independently of such tools, we will design the course so that for the most part you will not need to use a graphing calculator to do homework problems. Also, we will not

allow the use of calculators on the exams as this puts people with different models of calculators at a possible disadvantage. We will make sure that the problems on the tests require minimal calculation, to allow you to spend your time demonstrating your mathematical knowledge, not your calculating ability.