Math 21b midsemester exam guide

Regular sections

a) The exam

- ☐ The midsemester exam is on *Tuesday March 23* from *7-9pm* in *Science Center Lecture Halls A and D.* Go to either hall.
- ☐ Here is a facsimile of the first page of the exam booklet:

MATH 21b Midsemester Exam
S 2004
Regular sections

1) ___ 2) ___ 3) ___ 4) ___ 5) ___ Total ______

Name: ____

Circle the time of your section:

MWF10 • MWF11 • MWF12 • TTH10

Instructions:

- This exam booklet is only for students in the Regular sections.
- Print your name in the line above and circle the time of your section.
- Answer each of the questions below in the space provided. If more space is needed, use the back of the facing page or on the extra blank pages at the end of this booklet. Please direct the grader to the extra pages used.
- Please give justification for answers if you are not told otherwise.
- Please write neatly. Answers deemed illegible by the grader will not receive credit.
- No calculators, computers or other electronic aids are allowed; nor are you allowed to refer to any written notes or source material; nor are you allowed to communicate with other students. Use only your brain and a pencil.
- You have 1 and 1/2 hours to complete your work.

In agreeing to take this exam, you are implicitly agreeing to act with fairness and honesty.

- ☐ As you can see, there are five problems. The first consists of a series of true/false questions, while the remaining four are more or less of the sort that you have been doing in your homework assignments.
- ☐ The exam will cover the material in Chapters 1-3 and 5.1-5.3 in the text book, Linear Algebra and Applications.

- ☐ Advice for studying: There are plenty of answered problems in the text book and I strongly suggest that you work as many of these as you think necessary.
- ☐ Old exams: Old exams will not be terribly useful for the two reasons. First, in previous semesters, there were two midterm exams, not one. Thus, the exams from previous semesters will either test on material that we have not taught yet, or not test material that we have. Second, this version of Math 21b is <u>not</u> the same as those taught in previous semesters.
- ☐ Of the topics covered so far, some are more important than others. Given below is a list of those that are especially important.

b) Topics and issues to focus on

- ☐ Be able to the matrix that corresponds to a linear system of equations.
- ☐ Be able to rref(A) given the matrix A.
- ☐ Be able to solve $A\vec{x} = \vec{y}$ by computing rref for the augmented matrix, thus rref($A|\vec{y}$).
- ☐ Be able to find the inverse of a square matrix A by doing rref(A|I) where I is the identity matrix.
- ☐ Be able to use rref(A) to determine whether A is invertible, or if not, what its kernel is and what its image dimension is.
- ☐ Become comfortable with the notions that underlie the formal definitions of the following terms: linear transformation, linear subspace, the span of a set of vectors, linear dependence and linear independence, invertibility, orthogonality, kernel, image.
- ☐ Given a set, $\{\vec{v}_1, \ldots, \vec{v}_k\}$, of vectors in $\mathbb{R}^n$, be able to use the rref of the n-row/k-column matrix whose j'th column is $\vec{v}_j$ to determine if this set is linearly independent.
- ☐ Be able to find a basis for the kernel of a linear transformation.
- ☐ Be able to find a basis for the image of linear transformation.
- ☐ Know how to multiply matrices and also matrices against vectors. Know how these concepts respectively relate to the composition of two linear transformations and the action of a linear transformation.
- ☐ Know how the kernel and image of the product, AB, of matrices A and B are related to those of A and B.
- ☐ Understand how rref(AB) relates to rref(A) and rref(B).
- ☐ Be able to find the coordinates of a vector with respect to any given basis of $\mathbb{R}^n$.
- ☐ Be able to find the matrix of a linear transformation of $\mathbb{R}^n$ with respect to any given basis.
- ☐ Understand the relations between the triangle inequality ($|\vec{x}+\vec{y}| \le |\vec{x}| + |\vec{y}|$), the Cauchy-Schwarz inequality ($|\vec{x} \bullet \vec{y}| \le |\vec{x}||\vec{y}|$), and the Pythagorean equality ($|\vec{x}+\vec{y}|^2 = |\vec{x}|^2 + |\vec{y}|^2$).

- ☐ Be able to provide an orthonormal basis for a given linear subspace of $\mathbb{R}^n$. Thus, understand how to use the Gram-Schmidt procedure.
- ☐ Be able to give a matrix for the orthogonal projection of $\mathbb{R}^n$ onto any given linear subspace.
- ☐ Be able to work with the orthogonal complement of any given linear subspace in $\mathbb{R}^n$.
- ☐ Recognize that rotations are orthogonal transformations.
- ☐ Be able to recognize an orthogonal transformation: It preserves lengths. Such is the case if and only if its matrix, A, has the property that $|A\vec{x}| = |\vec{x}|$ for all vectors $\vec{x}$. Equivalent conditions: $A^{-1} = A^T$. Also, the columns of A form an orthonormal basis. Also, the rows of A form an orthonormal basis.
- ☐ Remember that the transpose of an orthogonal matrix is orthogonal, as is the product of any two orthogonal matrices.
- ☐ Be able to recognize symmetric and skew-symmetric matrices.