10. $f_A(\lambda) = (\lambda+1)^2(\lambda-1)$ so $\lambda_1 = -1$ (Algebraic multiplicity 2), $\lambda_2 = 1$.

11. $f_A(\lambda) = \lambda^3 + \lambda^2 + \lambda + 1 = (\lambda+1)(\lambda^2+1) = 0$
$\lambda = -1$ (Algebraic multiplicity 1).

12. $f_A(\lambda) = \lambda(\lambda+1)(\lambda-1)^2$ so $\lambda_1 = 0$, $\lambda_2 = -1$, $\lambda_3 = 1$ (Algebraic multiplicity 2).

16. $f_A(\lambda) = \lambda^2 - (a+c)\lambda + (ac - b^2)$

The discriminant of this quadratic equation is $(a+c)^2 - 4(ac-b^2) = a^2 + 2ac + c^2 - 4ac + 4b^2$ $= (a-c)^2 + 4b^2$; this quantity is always positive (since $b \neq 0$). There will always be two distinct real eigenvalues.

22. By Fact 6.2.1, $f_A(\lambda) = \det(\lambda I_n - A) = \det(\lambda I_n - A)^T = \det(\lambda I_n - A^T) = f_{A^T}(\lambda)$. Since the characteristic polynomials of A and A^T are identical, the two matrices have the same eigenvalues, with the same algebraic multiplicities.

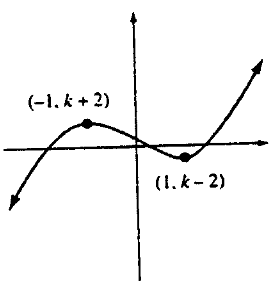
28. a. $w(t+1) = 0.8w(t) + 0.1m(t)$
$m(t+1) = 0.2w(t) + 0.9m(t)$
so $A = \begin{bmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{bmatrix}$ which is a regular transition matrix since its columns sum to 1 and its entries are positive.

b. The eigenvectors of A are $\begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}$ or $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ with $\lambda_1 = 1$, and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ with $\lambda_2 = 0.7$.

$\vec{x}_0 = \begin{bmatrix} 1200 \\ 0 \end{bmatrix} = 400\begin{bmatrix} 1 \\ 2 \end{bmatrix} + 800\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ so $\vec{x}(t) = 400\begin{bmatrix} 1 \\ 2 \end{bmatrix} + 800(0.7)^t\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ or
$w(t) = 400 + 800(0.7)^t$
$m(t) = 800 - 800(0.7)^t$.

c. As $t \to \infty$, $w(t) \to 400$ so Wipfs won't have to close the store.

32. $f_A(\lambda) = \lambda^3 - 3\lambda + k$ Setting $f'_A(\lambda) = 0$ we find that $f_A(\lambda)$ has a local minimum at $(1, k-2)$ and a local maximum at $(-1, k+2)$.



This polynomial has three distinct real roots if $k+2$ is positive (that is, if $k > -2$) and $k-2$ is negative (that is, $k < 2$). Both conditions are satisfied if $|k| < 2$. If $|k| = 2$, then there are two distinct roots; if $|k| > 2$, then there is only one real root.

36. Let $A = \begin{bmatrix} B & & 0 \\ & B & \\ & & \ddots & \\ 0 & & & B \end{bmatrix}$ where $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$; $f_A(\lambda) = (\lambda^2 + 1)^n$.