MATH 21B SOLUTIONS, CHAPTER 1 TRUE/FALSE

ALEXANDER ELLIS

2. F; Consider the equation $x + y + z = 0$ repeated four times.

4. T, by Definition 1.3.6.

6. T, by Fact 1.3.4.

8. F; As a counter-example, consider the zero matrix.

10. T, by Definition 1.3.7.

12. F; The product on the left-hand side has two components.

14. T; We have $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = 2 \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} - \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$.

16. T; $A = \begin{pmatrix} 3 & 0 \\ 4 & 0 \end{pmatrix}$, for example.

18. T;, find rref.

20. F; Note that $A \begin{pmatrix} 2 \\ 2 \end{pmatrix} = 2A \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ for all 2×2 matrices A.

22. T; Note that $\vec{v} = 1\vec{v} + 0\vec{w}$.

24. T; Note that $\vec{0} = 0\vec{v} + 0\vec{w}$.

26. T; If $\vec{u} = a\vec{v} + b\vec{w}$ and $\vec{v} = c\vec{p} + d\vec{q} + e\vec{r}$, then $\vec{u} = ac\vec{p} + ad\vec{q} + ae\vec{r} + b\vec{w}$.

28. F; Let $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, for example.

30. T, by Exercise 1.3.44.

32. T; Note that $\text{rank}(A) = 4$, by Fact 1.3.4.

34. T; we use rref to solve the system $A\vec{x} = \vec{0}$ and find $\vec{x} = \begin{pmatrix} -2t \\ -3t \\ t \end{pmatrix}$, where t is an arbitrary constant. Letting $t = 1$, we find $\begin{pmatrix} \vec{u} & \vec{v} & \vec{w} \end{pmatrix} \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix} = -2\vec{u} - 3\vec{v} + \vec{w} = \vec{0}$, so that $\vec{w} = 2\vec{u} + \vec{v}$.

36. T; Matrices A and B can both be transformed into $I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$. Running the elementary operations backwards, we can transform I into B. Thus we can first transform A into I and then I into B.

38. T; check that the three defining properties of a matrix in rref still hold.

40. F; If $\vec{b} = \vec{0}$, then having a row of zeroes in rref(A) does not force the system to be inconsistent.

42. T; The system $A\vec{x} = \vec{b}$ is consistent, by Example 3b, and there are, in fact, infinitely many solutions, by Fact 1.3.3. Note that $A\vec{x} = \vec{b}$ is a system of three equations with four unknowns.

44. F; Consider $\begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix}$. If we remove the first column, then the remaining matrix fails to be in rref.