# MATH 21B SOLUTIONS, CHAPTER 2 TRUE/FALSE

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**2.** F; the columns of a rotation matrix are unit vectors; see Fact 2.2.3.

**4.** T; Let $A = B$ in Fact 2.4.8

**6.** T, by Fact 2.4.9.

**8.** F; Note that $T\begin{pmatrix}0\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$. A linear transformation transforms $\vec{0}$ into $\vec{0}$.

**10.** T, by Fact 2.4.5.

**12.** T, by Fact 2.3.3.

**14.** T; Simplify to see that $T\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}4y\\-12x\end{pmatrix} = \begin{pmatrix}0 & 4\\-12 & 0\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$.

**16.** T; The matrix fails to be invertible for $k = 5$ and $k = -1$, since the determinant is 0 for these values.

**18.** T; Note that the columns are unit vectors, since $(-0.6)^2 = (\pm 0.8)^2 = 1$. The matrix has the form presented in Fact 2.2.3.

**20.** T; Note that $A = \begin{pmatrix}1 & 2\\3 & 4\end{pmatrix}^{-1}\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}\begin{pmatrix}5 & 6\\7 & 8\end{pmatrix}^{-1}$ is the unique solution.

**22.** T; One solution is $A = \begin{pmatrix}1 & 1\\0 & 0\end{pmatrix}$.

**24.** T; Just multiply it out.

**26.** T; Writing an upper triangular matrix $A = \begin{pmatrix}a & b\\0 & c\end{pmatrix}$ and solving the equation $A^2 = \begin{pmatrix}0 & 0\\0 & 0\end{pmatrix}$ we find that $A = \begin{pmatrix}0 & b\\0 & 0\end{pmatrix}$, where $b$ is any nonzero constant.

**28.** F; If a matrix $A$ is invertible, then so is $A^{-1}$. But $\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$ fails to be invertible.

**30.** T, by Fact 2.4.9. Note that $A^{-1} = A$ in this case.

**32.** F; Consider the reflection matrix $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

**34.** T; The equation $A\vec{e_i} = B\vec{e_i}$ means that the $i$th columns of $A$ and $B$ are identical. This observation applies to all the columns.

**36.** T; Multiply both sides of the equation $A^2 = A$ with $A^{-1}$.

**38.** T; Since $A\vec{x}$ is on the line onto which we project, the vector $A\vec{x}$ remains unchanged when we project again: $A(A\vec{x}) = A\vec{x}$, or $A^2\vec{x} = A\vec{x}$, for all $\vec{x}$. Thus $A^2 = A$.

**40.** T; Apply Fact 2.4.9 to the equation $(A^2)^{-1}AA = I_n$, with $B = (A^2)^{-1}A$.

**42.** F; Let $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, $\vec{v} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\vec{w} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

**44.** F; By Fact 1.3.3, there is a nonzero vector $\vec{x}$ such that $B\vec{x} = \vec{0}$, so that $AB\vec{x} = \vec{0}$ as well. But $I_3\vec{x} = \vec{x} \neq \vec{0}$, so that $AB \neq I_3$.

**46.** T; Note that $(I_n + A)(I_n - A) = I_n^2 - A^2 = I_n$, so that $(I_n + A)^{-1} = I_n - A$.

**48.** F; Consider $T(\vec{x}) = 2\vec{x}$, $\vec{v} = \vec{e_1}$, and $\vec{w} = \vec{e_2}$.

**50.** F; Let $A = B = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$, for example.

**52.** T; Consider an $\vec{x}$ such that $A^2\vec{x} = \vec{b}$, and let $\vec{x_0} = A\vec{x}$. Then $A\vec{x_0} = A(A\vec{x}) = A^2\vec{x} = \vec{b}$, as required.

**54.** F; Consider a $2 \times 2$ matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. We make an attempt to solve the equation

$$A^2 = \begin{pmatrix} a^2 + bc & ab + bd \\ ac + cd & cb + d^2 \end{pmatrix} = \begin{pmatrix} a^2 + bc & b(a+d) \\ c(a+d) & d^2 + bc \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Now the equation $b(a+d) = 0$ implies that $b = 0$ or $d = -a$. If $b = 0$, then the equation $d^2 + bc = -1$ cannot be solved. If $d = -a$, then the two diagonal entries of $A^2$, which are $a^2 + bc$ and $d^2 + bc$, will be equal, so that they cannot equal both 1 and $-1$. Thus, the equation $A^2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ cannot be solved.

**56.** T; We observe that the systems $AB\vec{x} = \vec{0}$ and $B\vec{x} = \vec{0}$ have the same solutions (multiply with $A^{-1}$ and $A$, respectively, to obtain one system from the other). Then, by True or False Exercise 45 in Chapter 1, $\text{rref}(AB) = \text{rref}(B)$.