MATH 21B SOLUTIONS, CHAPTER 3 TRUE/FALSE

ALEXANDER ELLIS

2. T; by Definition 3.1.2.

4. F, by Fact 3.3.7.

6. F; The identity matrix is similar only to itself.

8. F; The columns could be $\vec{e_1}, \vec{e_2}, \vec{e_3}, \vec{e_4}$ in $\mathbb{R}^5$, for example.

10. F; The nullity is $6 - 4 = 2$, Fact 3.3.7.

12. T, by Summary 3.3.9.

14. T, by Definition 3.2.1 (V is closed under linear combinations).

16. F; let $V = \text{span}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ in $\mathbb{R}^2$, for example.

18. T, by Definition 3.2.1.

20. T, by Fact 3.3.8.

22. F; Five vectors in $\mathbb{R}^4$ must be dependent, by Fact 3.2.8.

24. T; Use a basis with one vector on the line and the other perpendicular to it.

26. T, by Definition 3.2.3.

28. F; Consider $\vec{u} = \vec{e_1}$, $\vec{v} = 2\vec{e_1}$, and $\vec{w} = \vec{e_2}$.

30. T, since both kernels consist of the zero vector alone.

32. F; The identity matrix is similar only to itself.

34. F; Let $A = I_2$, $B = -I_2$, and $\vec{v} = \vec{e_1}$, for example.

36. T; If $A\vec{v} = A\vec{w}$, then $A(\vec{v} - \vec{w}) = \vec{0}$, so that $\vec{v} - \vec{w} = \vec{0}$ and thus $\vec{v} = \vec{w}$.

38. F; Suppose A were similar to B. Then $A^4 = I_2$ were similar to $B^4 = -I_2$, by Example 7 of Section 3.4. But this isn't the case: I_2 is similar only to itself.

40. T; If $B = S^{-1}AS$, then $B + 7I_n = S^{-1}(A + 7I_n)S$.

42. F; Consider I_n and $2I_n$, for example.

44. T; Note that $\text{im}(A)$ is a subspace of $\ker(A)$, so that

$$\dim(\text{im}(A)) = \text{rank}(A) \le \dim(\ker(A)) = 10 - \text{rank}(A)$$

46. T; check that $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ is similar to $\begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}$.

48. F; If $B = S^{-1}AS$, then $B = (2S)^{-1}A(2S)$ as well.

50. T; Suppose $\vec{v}$ is in both $\ker(A)$ and $\text{im}(A)$, so that $\vec{v} = A\vec{w}$ for some vector $\vec{w}$. Then $\vec{0} = A\vec{v} = A^2\vec{w} = A\vec{w} = \vec{v}$, as claimed.

52. T; the ith column $\vec{a_i}$ of A, being in the image of A, is also in the image of B, so that $\vec{a_i} = B\vec{c_i}$ for some $\vec{c_i}$ in $\mathbb{R}^m$. If we let $C = \begin{pmatrix} \vec{c_1} & \cdots & \vec{c_m} \end{pmatrix}$, then $BC = \begin{pmatrix} B\vec{c_1} & \cdots & B\vec{c_m} \end{pmatrix} = \begin{pmatrix} \vec{a_1} & \cdots & \vec{a_m} \end{pmatrix} = A$, as required.