MATH 21B SOLUTIONS, CHAPTER 5 TRUE/FALSE

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2. F. We have $(AB)^T = B^T A^T$, by Fact 5.3.9a.

4. T, by Fact 5.3.4.

6. T. First note that $A^T = A^{-1}$, by Fact 2.4.9. Thus A is orthogonal, by Fact 5.3.7.

8. T, since $(7A)^T = 7A^T = 7A$.

10. T, by Fact 5.3.9b.

12. T, since $A^T B^T = (BA)^T = (AB)^T = B^T A^T$, by Fact 5.3.9a.

14. T. Consider the QR factorization (Fact 5.2.2).

16. T. $\det \begin{pmatrix} a & c \\ b & d \end{pmatrix} = ad - bc = \det \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

18. F. Consider $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

20. F. It is required that the columns of A be orthonormal (Fact 5.3.10). As a counterexample, consider $A = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$, with $AA^T = \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix}$.

22. T. Use the Gram-Schmidt process to construct such a basis (Fact 5.2.1).

24. T, by definition of an orthogonal projection (Fact 5.1.4).

26. T, by Fact 5.4.1.

28. F. Consider $A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$, or any other symmetric matrix that fails to be orthogonal.

30. T.

$$\left[\frac{1}{2}(A - A^T)\right]^T = \frac{1}{2}(A - A^T)^T = \frac{1}{2}(A^T - A) = -\frac{1}{2}(A - A^T)$$

32. T. By Definition 5.1.12, quantity $\cos\theta = \frac{\vec{v}\cdot\vec{w}}{\|\vec{v}\|\|\vec{w}\|}$ is positive, so that θ is an acute angle.

34. T, since $(A^TA)^T = A^T(A^T)^T = A^TA$, by Fact 5.3.9a.

36. F. Consider $B = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$. The correct formula $\text{im}(B) = \text{im}(BB^T)$ follows from Facts 5.4.1 and 5.4.2.

38. T. By Fact 5.4.2, we have $\ker(A) = \ker(A^TA)$. Replacing A by A^T in this formula, we find that $\ker(A^T) = \ker(AA^T)$. Now $\ker(A) = \ker(A^TA) = \ker(AA^T) = \ker(A^T)$.

40. F. Consider $A = \begin{pmatrix} 0 & -1/2 \\ 1/2 & 0 \end{pmatrix}$, for example, representing a rotation combined with a scaling.

42. T. Apply the Cauchy-Schwarz inequality (squared), $(\vec{x} \cdot \vec{y})^2 \leq \|\vec{x}\|^2 \|\vec{y}\|^2$, to $\vec{x} = \begin{pmatrix} \vec{x_1} \\ \vdots \\ \vec{x_n} \end{pmatrix}$ and $\vec{y} = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$ (all n entries are 1).