Homework 1: Linear Equations

This homework is due on Monday, January 29, respectively Tuesday January 30, 2018. Homework is due at the beginning of each class in the classroom.

1 Find all solutions of the linear system
$$\begin{vmatrix} x + y + z + u & = 1 \\ x + y - u + v & = 2 \\ x + z & = 3 \\ x + y + u & = 4 \\ y + v & = 5 \end{vmatrix}$$

**2** On the iWatch one can play a 4×4 Suduku. The rules are that in each of the four 2×2 sub-squares, in each of the four rows and each of the four columns, the entries 1 to 4 have to appear and so add up to 10. The game initially gives 4 numbers. We have already started to enter 4 more numbers. There are still 8 missing. Introduce 8 variables and write down a system of 12 linear equations for these 8 variables, then solve the system.
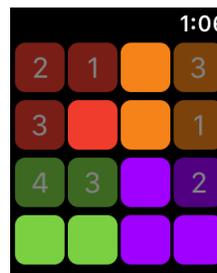
3 A 10 km trip from the Swiss waterfall "Rheinfall" to the village "Rheinau" takes 30 minutes. The return trip takes an hour. How fast is the speed v (in km/h) of the boat traveling relative to the water, and how fast is the speed s (in km/h) of the river?

4 On a heating mesh, the temperature at exterior mesh points is $0, 400$ or 800 F as given in the picture. In thermal equilibrium, each interior mesh point has the average of the temperatures at the 4 adjacent points. For example $T_2 = (T_3 + T_1 + 400 + 0)/4$. Find the temperatures T_1, T_2, T_3.
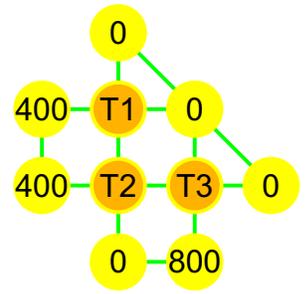
5 A polyhedron has v vertices, e edges and f triangular faces. Euler proved his famous formula $v - e + f = 2$ for such "discrete spheres". There is an other relation, $3f = 2e$ called a Dehn-Sommerville relation which always holds. Lets call this number f the "area". You get a polyhedron with area f=288. Write down a system of equations in matrix form $Ax = b$. Then determine the number of vertices and edges.
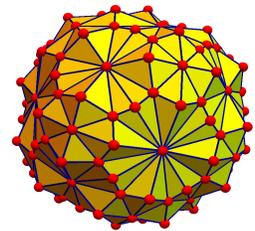
Main definitions

A **linear equation** for variables $x_1, x_2, \ldots, x_n$ is an equation $a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$. Given m equations of this type, we get a **system of linear equations**. It can be written in matrix form $A\vec{x} = \vec{b}$, where $\vec{x}$ is a column vector containing the n variables and the $m \times n$ matrix A lists the $m \cdot n$ coefficients and $\vec{b}$ is the column vector. The system $x + 2y + z = 8, 3x - y - 7z = 4$ for example can be written as

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & -1 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \end{bmatrix}.$$