Homework 11: Linear spaces

This homework [1] is due on Friday, February 23, respectively on Thursday, February 22, 2018.

1 Which of the following spaces are linear spaces?

a) All 4×4 matrices for which the diagonal entries are all zero.

b) All 4×4 matrices for which the product of all matrix entries is zero.

c) All polynomials of degree exactly 4.

d) All 2×2 projection dilation matrices $A = \begin{bmatrix} a^2 & ab \\ ab & b^2 \end{bmatrix}$.

e) All 2×2 matrices A for which $A^2 = 0$.

f) All functions f in $C(R)$ for which $f(5) = 0$.

g) All functions f in $C^\infty(R)$ for which $f''(0) = f'(1)$.

h) All functions f in $C^\infty(R)$ for which $f(0)^2 + f''(2) = 0$.

2 Find a basis for the space of

a) all 2×2 rotation dilation matrices $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$.

b) all 2×2 reflection dilation matrices $\begin{bmatrix} a & b \\ b & -a \end{bmatrix}$.

c) all 2×2 horizontal shears. (Trick question! Why?)

d) all the 2×2 matrices for which Ae_1 is parallel to e_1.

e) all the diagonal 3×3 matrices.

f) all the 2×2 dilation matrices.

3 Find a basis for all the 2×2 matrices A for which

$$A \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

[1] While not much seems to happen in this lecture we actually make a giant step. The language of linear algebra is pushed into realms where it originally was not used like to spaces of functions used in calculus. You can now look at a spread sheet or picture or movie or a table in a relational database and see it as a vector in a linear space.

4 A function is called **even** if $f(-x) = f(x)$ for all x. A function is called **odd** if $f(-x) = -f(x)$ for all x. Find a basis:
a) for all the even polynomials in $P_4(R)$
b) for all the odd polynomials in $P_4(R)$.
c) for all polynomials f in $P_4(R)$ for which $f(0) = 0$.

5 a) Find a concrete 3×3 matrix A with no zeros for which $A\vec{v} = \vec{0}$, where $\vec{v} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}^T$.
b) Let V be the set of 3×3 matrices for which $\vec{v}$ is in the kernel. Is this a linear space?

Linear space

A set V in which we can add, scale and which contains a 0 is a **linear space**. To check that V is a linear space, verify (i) 0 is in X, (ii) if x and y are both in X, then $x+y$ is in X, (iii) if x is in X, then λx is in X for all constants c. There are three important classes of linear spaces: first the subspaces of R^n as treated before, the second class is the set $M(n,m)$ of all $n \times m$ matrices. Finally, there is the class $C(R)$ for all continuous functions on the real line. It contains the linear space $C^\infty(R)$ for all smooth functions, functions which can be differentiated arbitrary often. We also write $P_n(R)$ for the set of polynomials of degree less or equal to n. It has dimension $n+1$. In order to check whether a subset of functions or matrices or vectors in R^n are a linear space, check the three things (i),(ii),(iii). In the case of functions, 0 is the function which is 0 for all x.