Math S-21b – Summer 2003 – Practice Exam #1

(1) True or False. (Circle one) You need not give your reasoning.

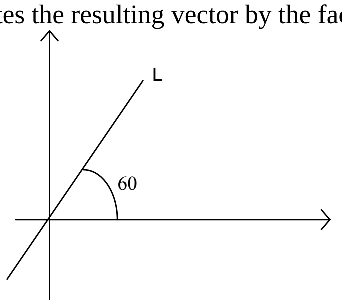
(a) Consider a system $\mathbf{Ax} = \mathbf{b}$. This system is consistent if and only if rank($\mathbf{A}$) = rank([$\mathbf{A}$|$\mathbf{b}$]), where [$\mathbf{A}$|$\mathbf{b}$] denotes the augmented matrix.	(a) TRUE FALSE
(b) If $\mathbf{A}$ and $\mathbf{B}$ are n x n matrices such that the kernel of $\mathbf{A}$ is contained in the image of $\mathbf{B}$, then the matrix $\mathbf{AB}$ cannot be invertible.	(b) TRUE FALSE
(c) If $\mathbf{A}$ is an 8×5 matrix, then the kernel of $\mathbf{A}$ is at least three-dimensional.	(c) TRUE FALSE
(d) Consider the vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ in $\mathbf{R}^m$. Let $\mathbf{A}$ be a $p \times m$ matrix. If the vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ are linearly dependent, then so are the vectors $\mathbf{Av}_1, \mathbf{Av}_2, \ldots, \mathbf{Av}_n$.	(d) TRUE FALSE
(e) rank($\mathbf{A}^2$) $\leq$ rank($\mathbf{A}$) for any square matrix $\mathbf{A}$. ($\mathbf{A}^2$ denotes the product **AA.)**	(e) TRUE FALSE
(f) There is a 4 x 4 matrix $\mathbf{A}$ such that image($\mathbf{A}$) and kernel($\mathbf{A}$) are the same subspace of $\mathbf{R}^4$.	(f) TRUE FALSE

(2) Let $\mathbf{A}$ be the 4 x 5 matrix $\mathbf{A} = \begin{bmatrix} 1 & -2 & 1 & 5 & 0 \\ 1 & -2 & -1 & 1 & 0 \\ 3 & -6 & -1 & 7 & -1 \\ -1 & 2 & 0 & -3 & 0 \end{bmatrix}$.

(a) Find a basis for the kernel of $\mathbf{A}$ and its dimension.
(b) Find a basis for the image of $\mathbf{A}$ and its dimension.

(c) Find all solutions of the equation $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{b} = \begin{bmatrix} 3 \\ 5 \\ 1 \\ -4 \end{bmatrix}$.

(3) Let T($\mathbf{x}$) = $\mathbf{Ax}$ be the linear transformation from $\mathbf{R}^2$ to $\mathbf{R}^2$ which first reflects a vector in the line L shown below and then dilates the resulting vector by the factor 3.



(a) Find the matrix $\mathbf{A}$.

(b) Find $\mathbf{A}^4 \begin{bmatrix} -4 \\ 1 \end{bmatrix}$.

($\mathbf{A}^4$ denotes the product **AAAA.)**

(4) A matrix can described geometrically by how it acts on any basis of vectors.

(a) Given the vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ where $\mathbf{v}_1 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$, show that these three vectors are linearly independent (and are therefore a basis for $\mathbf{R}^3$).

(b) If we know that $\mathbf{Av}_1 = \mathbf{v}_1 - 2\mathbf{v}_2 + \mathbf{v}_3$, $\mathbf{Av}_2 = -2\mathbf{v}_1 + \mathbf{v}_2 - \mathbf{v}_3$, and $\mathbf{Av}_3 = \mathbf{v}_1 + \mathbf{v}_2 - 3\mathbf{v}_3$, determine the matrix $\mathbf{A}$.