2.4

1. $\begin{bmatrix} 4 & 6 \\ 3 & 4 \end{bmatrix}$

2. $\begin{bmatrix} 4 & 4 \\ -8 & -8 \end{bmatrix}$

3. Undefined.

4. $\begin{bmatrix} 2 & 2 \\ 2 & 0 \\ 7 & 4 \end{bmatrix}$

5. $\begin{bmatrix} a & b \\ c & d \\ 0 & 0 \end{bmatrix}$

6. $\begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$

7. $\begin{bmatrix} -1 & 1 & 0 \\ 5 & 3 & 4 \\ -6 & -2 & -4 \end{bmatrix}$

8. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

9. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

10. $[0\ 1]$

11. $[10]$

12. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$

13. $[h]$

14. $A^2 = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$, $BC = [14\ \ 8\ \ 2]$, $BD = [6]$, $C^2 = \begin{bmatrix} -2 & -2 & -2 \\ 4 & 1 & -2 \\ 10 & 4 & -2 \end{bmatrix}$, $CD = \begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix}$, $DB = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$, $DE = \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}$, $EB = [5\ 10\ 15]$, $E^2 = [25]$

15. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; Fact 2.4.9 applies to square matrices only.

16. True; $(I_n - A)(I_n + A) = I_n^2 + A - A - A^2 = I_n - A^2$.

17. Not necessarily true; $(A+B)^2 = (A+B)(A+B) = A^2 + AB + BA + B^2 \neq A^2 + 2AB + B^2$ if $AB \neq BA$.

18. True; apply Fact 2.4.8 to $B = A$.

19. Not necessarily true; consider the case $A = I_n$ and $B = -I_n$.

20. Not necessarily true; $(A-B)(A+B) = A^2 + AB - BA - B^2 \neq A^2 - B^2$ if $AB \neq BA$.

21. True; $ABB^{-1}A^{-1} = AI_nA^{-1} = AA^{-1} = I_n$.

22. Not necessarily true; the equation $ABA^{-1} = B$ is equivalent to $AB = BA$ (multiply by A from the right), which is not true in general.

23. True; $(ABA^{-1})^3 = ABA^{-1}ABA^{-1}ABA^{-1} = AB^3A^{-1}$.

24. True; $(I_n + A)(I_n + A^{-1}) = I_n^2 + A + A^{-1} + AA^{-1} = 2I_n + A + A^{-1}$.

25. True; $(A^{-1}B)^{-1} = B^{-1}(A^{-1})^{-1} = B^{-1}A$ (use Fact 2.4.8).

43. Let A represent the rotation through 120°; then A^3 represents the rotation through 360°, that is $A^3 = I_2$.

$$A = \begin{bmatrix} \cos(120^\circ) & -\sin(120^\circ) \\ \sin(120^\circ) & \cos(120^\circ) \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$$

44. We want A such that $A\begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$, so that $A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}^{-1} = \begin{bmatrix} 8 & -3 \\ -1 & 1 \end{bmatrix}$.

45. We want A such that $A\vec{v}_i = \vec{w}_i$, for $i = 1, 2, \ldots, n$, or $A[\vec{v}_1 \ \vec{v}_2 \ \ldots \ \vec{v}_n] = [\vec{w}_1 \ \vec{w}_2 \ \ldots \ \vec{w}_n]$, or $AS = B$. Multiplying by S^{-1} from the right we find the unique solution $A = BS^{-1}$.

46. Use the result of Exercise 45, with $S = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 1 \\ 5 & 2 \\ 3 & 3 \end{bmatrix}$;

$$A = BS^{-1} = \begin{bmatrix} 33 & -13 \\ 21 & -8 \\ 9 & -3 \end{bmatrix}$$