5. $\vec{w}_1 = \frac{1}{\|\vec{v}_1\|}\vec{v}_1 = \frac{1}{3}\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$

$$\vec{w}_2 = \frac{\vec{v}_2 - (\vec{w}_1 \cdot \vec{v}_2)\vec{w}_1}{\text{length}} = \frac{1}{\sqrt{18}}\begin{bmatrix} -1 \\ -1 \\ 4 \end{bmatrix} = \frac{1}{3\sqrt{2}}\begin{bmatrix} -1 \\ -1 \\ 4 \end{bmatrix}$$

6. $\vec{w}_1 = \frac{1}{\|\vec{v}_1\|}\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \vec{e}_1$

$$\vec{w}_2 = \frac{\vec{v}_2 - (\vec{w}_1 \cdot \vec{v}_2)\vec{w}_1}{\text{length}} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \vec{e}_2$$

$$\vec{w}_3 = \frac{\vec{v}_3 - (\vec{w}_1 \cdot \vec{v}_3)\vec{w}_1 - (\vec{w}_2 \cdot \vec{v}_3)\vec{w}_2}{\text{length}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \vec{e}_3$$

7. Note that $\vec{v}_1$ and $\vec{v}_2$ are orthogonal, so that $\vec{w}_1 = \frac{1}{\|\vec{v}_1\|}\vec{v}_1 = \frac{1}{3}\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$ and $\vec{w}_2 = \frac{1}{\|\vec{v}_2\|}\vec{v}_2 = \frac{1}{3}\begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$. Then

$$\vec{w}_3 = \frac{\vec{v}_3 - (\vec{w}_1 \cdot \vec{v}_3)\vec{w}_1 - (\vec{w}_2 \cdot \vec{v}_3)\vec{w}_2}{\text{length}} = \frac{1}{\sqrt{36}}\begin{bmatrix} 2 \\ -4 \\ 4 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}.$$

8. $\vec{w}_1 = \frac{1}{\|\vec{v}_1\|}\vec{v}_1 = \frac{1}{7}\begin{bmatrix} 5 \\ 4 \\ 2 \\ 2 \end{bmatrix}$

$$\vec{w}_2 = \frac{\vec{v}_2 - (\vec{w}_1 \cdot \vec{v}_2)\vec{w}_1}{\text{length}} = \frac{1}{7}\begin{bmatrix} -2 \\ 2 \\ 5 \\ -4 \end{bmatrix}$$

13. $\vec{w}_1 = \frac{1}{\|\vec{v}_1\|}\vec{v}_1 = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

$$\vec{w}_2 = \frac{\vec{v}_2 - (\vec{w}_1 \cdot \vec{v}_2)\vec{w}_1}{\text{length}} = \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

$$\vec{w}_3 = \frac{\vec{v}_3 - (\vec{w}_1 \cdot \vec{v}_3)\vec{w}_1 - (\vec{w}_2 \cdot \vec{v}_3)\vec{w}_2}{\text{length}} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}$$

14. $\vec{w}_1 = \frac{1}{\|\vec{v}_1\|}\vec{v}_1 = \frac{1}{10}\begin{bmatrix} 1 \\ 7 \\ 1 \\ 7 \end{bmatrix}$

$$\vec{w}_2 = \frac{\vec{v}_2 - (\vec{w}_1 \cdot \vec{v}_2)\vec{w}_1}{\text{length}} = \frac{1}{\sqrt{2}}\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\vec{w}_3 = \frac{\vec{v}_3 - (\vec{w}_1 \cdot \vec{v}_3)\vec{w}_1 - (\vec{w}_2 \cdot \vec{v}_3)\vec{w}_2}{\text{length}} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

20. $Q = I_3$, $R = \begin{bmatrix} \vec{v}_1 & \vec{v}_2 & \vec{v}_3 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 5 \\ 0 & 4 & 6 \\ 0 & 0 & 7 \end{bmatrix}$

21. $Q = \frac{1}{3}\begin{bmatrix} 2 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 2 \end{bmatrix}$, $R = \begin{bmatrix} 3 & 0 & 12 \\ 0 & 3 & -12 \\ 0 & 0 & 6 \end{bmatrix}$

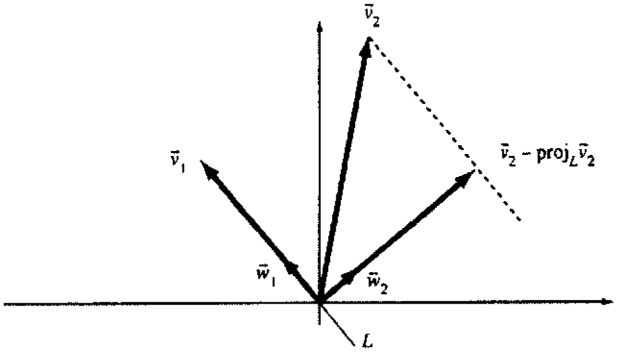
22. $Q = \frac{1}{7}\begin{bmatrix} 5 & -2 \\ 4 & 2 \\ 2 & 5 \\ 2 & -4 \end{bmatrix}$, $R = \begin{bmatrix} 7 & 7 \\ 0 & 7 \end{bmatrix}$

23. $Q = \begin{bmatrix} 0.5 & -0.1 \\ 0.5 & 0.7 \\ 0.5 & -0.7 \\ 0.5 & 0.1 \end{bmatrix}$, $R = \begin{bmatrix} 2 & 4 \\ 0 & 10 \end{bmatrix}$

28. $Q = \begin{bmatrix} \frac{1}{10} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{7}{10} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{10} & \frac{1}{\sqrt{2}} & 0 \\ \frac{7}{10} & 0 & -\frac{1}{\sqrt{2}} \end{bmatrix}$, $R = \begin{bmatrix} 10 & 10 & 10 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix}$

29. $\vec{w}_1 = \frac{1}{\|\vec{v}_1\|}\vec{v}_1 = \frac{1}{5}\begin{bmatrix} -3 \\ 4 \end{bmatrix}$

$$\vec{w}_2 = \frac{\vec{v}_2 - (\vec{w}_1 \cdot \vec{v}_2)\vec{w}_1}{\text{length}} = \frac{1}{5}\begin{bmatrix} 4 \\ 3 \end{bmatrix}$$



34. $\text{rref}(A) = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \end{bmatrix}$

A basis of $\ker(A)$ is $\vec{v}_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 2 \\ -3 \\ 0 \\ 1 \end{bmatrix}$.

We apply the Gram-Schmidt process and obtain

$$\vec{w}_1 = \frac{1}{\|\vec{v}_1\|}\vec{v}_1 = \frac{1}{\sqrt{6}}\begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{w}_2 = \frac{\vec{v}_2 - (\vec{w}_1 \cdot \vec{v}_2)\vec{w}_1}{\text{length}} = \frac{1}{\sqrt{30}}\begin{bmatrix} 2 \\ -1 \\ -4 \\ 3 \end{bmatrix}$$

35. $\text{rref}(A) = \begin{bmatrix} 1 & 0 & \frac{1}{3} \\ 0 & 1 & \frac{1}{3} \\ 0 & 0 & 0 \end{bmatrix}$

The pivot columns of A give us a basis of $\text{im}(A)$:

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

Since $\vec{v}_1$ and $\vec{v}_2$ are orthogonal already, we obtain $\vec{w}_1 = \frac{1}{3}\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$, $\vec{w}_2 = \frac{1}{3}\begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$.