15. Any vector on L is unaffected by the reflection, so that a nonzero vector on L is an eigenvector with eigenvalue 1. Any vector on $L^\perp$ is flipped about L, so that a nonzero vector on $L^\perp$ is an eigenvector with eigenvalue -1. Picking a nonzero vector from L and one from $L^\perp$, we obtain a basis consisting of eigenvectors.

16. Rotation by 180° is a flip about the origin so every nonzero vector is an eigenvector with the eigenvalue -1. Any basis for $\mathbb{R}^2$ consists of eigenvectors.

17. No (real) eigenvalues

18. Any nonzero vector in the plane is unchanged, hence is an eigenvector with the eigenvalue 1. Since any nonzero vector in $E^\perp$ is flipped about the origin, it is an eigenvector with eigenvalue -1. Pick any two non-collinear vectors from E and one from $E^\perp$ to form a basis consisting of eigenvectors.

19. Any nonzero vector in L is an eigenvector with eigenvalue 1, and any nonzero vector in the plane $L^\perp$ is an eigenvector with eigenvalue 0. Form an eigenbasis by picking any nonzero vector in L and any two noncollinear vectors in $L^\perp$.

20. Any nonzero vector along the $\vec{e}_3$-axis is unchanged, hence is an eigenvector with eigenvalue 1. No other (real) eigenvalues can be found.

21. Any nonzero vector in $\mathbb{R}^3$ is an eigenvector with eigenvalue 5. Any basis for $\mathbb{R}^3$ consists of eigenvectors.

34. We want A such that $A\begin{bmatrix}3\\1\end{bmatrix}=\begin{bmatrix}15\\5\end{bmatrix}$ and $A\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}10\\20\end{bmatrix}$, i.e. $A\begin{bmatrix}3&1\\1&2\end{bmatrix}=\begin{bmatrix}15&10\\5&20\end{bmatrix}$, so

$$A=\begin{bmatrix}15&10\\5&20\end{bmatrix}\begin{bmatrix}3&1\\1&2\end{bmatrix}^{-1}=\begin{bmatrix}4&3\\-2&11\end{bmatrix}.$$

39. Let λ be an eigenvalue of $S^{-1}AS$. Then for some nonzero vector $\vec{v}$, $S^{-1}AS\vec{v} = \lambda\vec{v}$, i.e., $AS\vec{v} = S\lambda\vec{v} = \lambda S\vec{v}$ so λ is an eigenvalue of A with eigenvector $S\vec{v}$.
Conversely, if α is an eigenvalue of A with eigenvector $\vec{w}$, then $A\vec{w} = \alpha\vec{w}$.
Therefore, $S^{-1}AS(S^{-1}\vec{w}) = S^{-1}A\vec{w} = S^{-1}\alpha\vec{w} = \alpha S^{-1}\vec{w}$, so $S^{-1}\vec{w}$ is an eigenvector of $S^{-1}AS$ with eigenvalue α.