2. $\lambda_1 = 2, \lambda_2 = 0, E_2 = \text{span}\begin{bmatrix}1\\1\end{bmatrix}, E_0 = \text{span}\begin{bmatrix}-1\\1\end{bmatrix}$

   Eigenbasis: $\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}-1\\1\end{bmatrix}$

3. $\lambda_1 = 4,\ \lambda_2 = 9,\ E_4 = \text{span}\begin{bmatrix}3\\-2\end{bmatrix},\ E_9 = \text{span}\begin{bmatrix}1\\1\end{bmatrix}$

   Eigenbasis: $\begin{bmatrix}3\\-2\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}$

4. $\lambda_1 = \lambda_2 = 1, E_1 = \text{span}\begin{bmatrix}-1\\1\end{bmatrix}$

   No eigenbasis

5. No real eigenvalues as $f_A(\lambda) = \lambda^2 - 2\lambda + 2$.

6. $\lambda_{1,2} = \dfrac{7 \pm \sqrt{57}}{2}$

   Eigenbasis: $\begin{bmatrix}3\\\lambda_1 - 2\end{bmatrix} \approx \begin{bmatrix}3\\5.27\end{bmatrix}, \begin{bmatrix}3\\\lambda_2 - 2\end{bmatrix} \approx \begin{bmatrix}3\\-2.27\end{bmatrix}$

7. $\lambda_1 = 1,\ \lambda_2 = 2,\ \lambda_3 = 3$, eigenbasis: $\vec{e}_1,\ \vec{e}_2,\ \vec{e}_3$

8. $\lambda_1 = 1,\ \lambda_2 = 2,\ \lambda_3 = 3$, eigenbasis: $\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\2\\1\end{bmatrix}$

18. $\lambda_1 = \lambda_2 = 0$, $\lambda_3 = \lambda_4 = 1$, $E_0 = \text{span}(\vec{e}_1, \vec{e}_3)$, $E_1 = \text{span}(\vec{e}_2)$
No eigenbasis

44. a. $a_{11} = 0.7$ means that only 70% of the pollutant present in Lake Silvaplana at a given time is still there a week later; some is carried down to Lake Sils by the river Inn, and some is absorbed or evaporates.

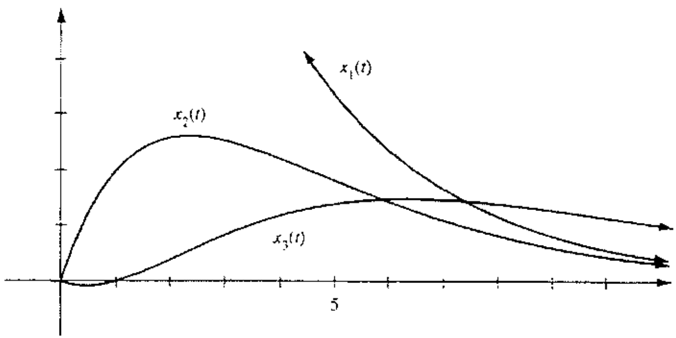
The other diagonal entries can be interpreted analogously. $a_{21} = 0.1$ means that 10% of the pollutant present in Lake Silvaplana at any given time can be found in Lake Sils a week later, carried down by the river Inn. The significance of the coefficient $a_{32} = 0.2$ is analogous; $a_{31} = 0$ means that no pollutant is carried down from Lake Silvaplana to Lake St. Moritz in just one week.

The matrix is lower triangular since no pollutant is carried from Lake Sils to Lake Silvaplana, for example (the river Inn flows the other way).

b. The eigenvalues of A are 0.8, 0.6, 0.7 with corresponding eigenvectors $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$.

$\vec{x}(0) = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix} = 100 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - 100 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} + 100 \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$ so $\vec{x}(t) = 100(0.8)^t \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - 100(0.6)^t \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} +$ $100(0.7)^t \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$ or

$x_1(t) = 100(0.7)^t$
$x_2(t) = 100(0.7)^t - 100(0.6)^t$
$x_3(t) = 100(0.8)^t + 100(0.6)^t - 200(0.7)^t$



Using calculus, we find that the function $x_2(t) = 100(0.7)^t - 100(0.6)^t$ reaches its maximum at $t \approx 2.33$. Keep in mind, however, that our model holds for integer t only.