2. The solutions of $\frac{dx}{dt} + 3x = 0$ are of the form $x(t) = Ce^{-3t}$, where C is an arbitrary constant, and the differential equation $\frac{dx}{dt} + 3x = 7$ has the particular solution $x_p(t) = \frac{7}{3}$, so that the general solution is $x(t) = Ce^{-3t} + \frac{7}{3}$ (where C is a constant). Alternatively, we could use Fact 9.3.13.

3. Use Fact 9.3.13, where $a = -2$ and $g(t) = e^{3t}$:
$f(t) = e^{-2t}\int e^{2t}e^{3t}\,dt = e^{-2t}\int e^{5t}\,dt = e^{-2t}\left(\frac{1}{5}e^{5t} + C\right) = \frac{1}{5}e^{3t} + Ce^{-2t}$, where C is a constant.

4. We can look for a sinusoidal solution $x_p(t) = P\cos(3t) + Q\sin(3t)$, as in Example 7. P and Q need to be chosen in such a way that $-3P\sin(3t) + 3Q\cos(3t) - 2P\cos(3t) - 2Q\sin(3t) = \cos(3t)$ or $\left|\begin{array}{l} -2P + 3Q = 1 \\ -3P - 2Q = 0 \end{array}\right|$ with solution $P = -\frac{2}{13}$ and $Q = \frac{3}{13}$. Since the general solution of $\frac{dx}{dt} - 2x = 0$ is $x(t) = Ce^{2t}$, the general solution of $\frac{dx}{dt} - 2x = \cos(3t)$ is $x(t) = Ce^{2t} - \frac{2}{13}\cos(3t) + \frac{3}{13}\sin(3t)$, where C is an arbitrary constant.

5. Using Fact 9.3.13, $f(t) = e^{t}\int e^{-t}t\,dt = e^{t}(-te^{-t} - e^{-t} + C) = Ce^{t} - t - 1$, where C is an arbitrary constant.

6. Using Fact 9.3.13, $f(t) = e^{2t}\int e^{-2t}e^{2t}\,dt = e^{2t}\int dt = e^{2t}(t + C)$, where C is an arbitrary constant.

7. By Definition 9.3.6, $p_T(\lambda) = \lambda^2 + \lambda - 12 = (\lambda + 4)(\lambda - 3)$.
Since $p_T(\lambda)$ has distinct roots $\lambda_1 = -4$ and $\lambda_2 = 3$, the solutions of the differential equation are of the form $f(t) = c_1e^{-4t} + c_2e^{3t}$, where c_1 and c_2 are arbitrary constants (by Fact 9.3.8).

13. $p_T(\lambda) = \lambda^2 + 2\lambda + 1 = (\lambda+1)^2 = 0$ has the double root $\lambda = -1$. Following the strategy on page 436, we find $f(t) = e^{-t}(c_1 t + c_2)$, where c_1, c_2 are arbitrary constants.

14. $p_T(\lambda) = \lambda^2 + 3\lambda = \lambda(\lambda+3) = 0$ has roots $\lambda_1 = 0, \lambda_2 = -3$.
$f(t) = c_1 e^{0t} + c_2 e^{-3t} = c_1 + c_2 e^{-3t}$, where c_1, c_2 are arbitrary constants.

15. By integrating twice we find $f(t) = c_1 + c_2 t$, where c_1, c_2 are arbitrary constants.

16. By Fact 9.3.10, the differential equation has a particular solution of the form $f_p(t) = P\cos(t) + Q\sin(t)$. Plugging f_p into the equation we find
$(-P\cos(t) - Q\sin(t)) + 4(-P\sin(t) + Q\cos(t)) + 13(P\cos(t) + Q\sin(t)) = \cos(t)$ or
$\left| \begin{array}{c} 12P + 4Q = 1 \\ -4P + 12Q = 0 \end{array} \right|$, so
$P = \frac{3}{40}$
$Q = \frac{1}{40}$.
Therefore, $f_p(t) = \frac{3}{40}\cos(t) + \frac{1}{40}\sin(t)$.
Next we find a basis of the solution space of $f''(t) + 4f'(t) + 13f(t) = 0$. $p_T(\lambda) = \lambda^2 + 4\lambda + 13 = 0$ has roots $-2 \pm 3i$. By Fact 9.3.9, $f_1(t) = e^{-2t}\cos(3t)$ and $f_2(t) = e^{-2t}\sin(3t)$ is a basis of the solution space.
By Fact 9.3.4, the solutions of the original differential equation are of the form $f(t) = c_1 f_1(t) + c_2 f_2(t) + f_p(t) = c_1 e^{-2t}\cos(3t) + c_2 e^{-2t}\sin(3t) + \frac{3}{40}\cos(t) + \frac{1}{40}\sin(t)$, where c_1, c_2 are arbitrary constants.

17. By Fact 9.3.10, the differential equation has a particular solution of the form $f_p(t) = P\cos(t) + Q\sin(t)$. Plugging f_p into the equation we find $(-P\cos(t) - Q\sin(t)) + 2(-P\sin(t) + Q\cos(t)) + P\cos(t) + Q\sin(t) = \sin(t)$ or $\left| \begin{array}{c} 2Q = 0 \\ -2P = 1 \end{array} \right|$, so $\begin{array}{c} P = -\frac{1}{2} \\ Q = 0 \end{array}$.
Therefore, $f_p(t) = -\frac{1}{2}\cos(t)$.
Next we find a basis of the solution space of $f''(t) + 2f'(t) + f(t) = 0$. In Exercise 13 we see that $f_1(t) = e^{-t}$, $f_2(t) = te^{-t}$ is such a basis.
By Fact 9.3.4, the solutions of the original differential equation are of the form $f(t) = c_1 f_1(t) + c_2 f_2(t) + f_p(t) = c_1 e^{-t} + c_2 t e^{-t} - \frac{1}{2}\cos(t)$, where c_1, c_2 are arbitrary constants.

24. General solution $x(t) = Ce^{-3t} + \frac{7}{3}$ (Exercise 2)
Plug in: $0 = x(0) = C + \frac{7}{3}$, so that $C = -\frac{7}{3}$ and $x(t) = -\frac{7}{3}e^{-3t} + \frac{7}{3}$.

25. General solution $f(t) = Ce^{-2t}$
Plug in: $1 = f(1) = Ce^{-2}$, so that $C = e^2$ and $f(t) = e^2e^{-2t} = e^{2-2t}$.

26. General solution $f(t) = c_1e^{3t} + c_2e^{-3t}$ (Exercise 9), with $f'(t) = 3c_1e^{3t} - 3c_2e^{-3t}$
Plug in: $0 = f(0) = c_1 + c_2$ and $1 = f'(0) = 3c_1 - 3c_2$, so that $c_1 = \frac{1}{6}, c_2 = -\frac{1}{6}$, and $f(t) = \frac{1}{6}e^{3t} - \frac{1}{6}e^{-3t}$.

27. General solution $f(t) = c_1\cos(3t) + c_2\sin(3t)$ (Fact 9.3.9)
Plug in: $0 = f(0) = c_1$ and $1 = f\left(\frac{\pi}{2}\right) = -c_2$, so that $c_1 = 0, c_2 = -1$, and $f(t) = -\sin(3t)$.

28. General solution $f(t) = c_1e^{-4t} + c_2e^{3t}$, with $f'(t) = -4c_1e^{-4t} + 3c_2e^{3t}$
Plug in: $0 = f(0) = c_1 + c_2$ and $0 = f'(0) = -4c_1 + 3c_2$, so that $c_1 = c_2 = 0$ and $f(t) = 0$.

29. General solution $f(t) = c_1\cos(2t) + c_2\sin(2t) + \frac{1}{3}\sin(t)$, so that $f'(t) = -2c_1\sin(2t) + 2c_2\cos(2t) + \frac{1}{3}\cos(t)$ (use the approach outlined in Exercises 16 and 17)
Plug in: $0 = f(0) = c_1$ and $0 = f'(0) = 2c_2 + \frac{1}{3}$, so that $c_1 = 0$, $c_2 = -\frac{1}{6}$, and $f(t) = -\frac{1}{6}\sin(2t) + \frac{1}{3}\sin(t)$.

30. a. k is a positive constant that depends on the rate of cooling of the coffee (it varies with the material of the cup, for example).
A is the room temperature.

b. $T'(t) + kT(t) = kA$
Constant particular solution: $T_p(t) = A$

General solution of $T'(t) + kT(t) = 0$ is $T(t) = Ce^{-kt}$.
General solution of the original differential equation: $T(t) = Ce^{-kt} + A$
Plug in: $T_0 = T(0) = C + A$, so that $C = T_0 - A$ and $T(t) = (T_0 - A)e^{-kt} + A$.

43. a. Using the approach of Exercises 16 and 17 we find $x(t) = c_1 e^{-2t} + c_2 e^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$.

b. For large t, $x(t) \approx \frac{1}{10}\cos t + \frac{1}{10}\sin t$.