True or False

1. F; It's a subspace of $\mathbb{R}^3$.

2. T; by Definition 3.1.2.

3. T, by Summary 3.3.11.

4. F, by Fact 3.3.9.

5. T, by Summary 3.3.11.

6. F; The identity matrix is similar only to itself.

7. T; We have the nontrivial relation $3\vec{u} + 3\vec{v} + 3\vec{w} = \vec{0}$.

8. F; The columns could be $\vec{e}_1, \vec{e}_2, \vec{e}_3, \vec{e}_4$ in $\mathbb{R}^5$, for example.

9. T, by Fact 3.3.2.

10. F; The nullity is $6 - 4 = 2$, by Fact 3.3.9.

11. T, by Fact 3.2.6.

12. T, by Summary 3.3.11.

13. F; The number n may exceed 4.

14. T, by Definition 3.2.1 (V is closed under linear combinations)

15. T, by Fact 3.4.6, parts b and c.

16. F; Let $V = \text{span}\begin{bmatrix}1\\1\end{bmatrix}$ in $\mathbb{R}^2$, for example.

17. T, by Definition 3.2.3.

18. T, by Definition 3.2.1.

19. T; Check that $\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}\begin{bmatrix}1 & 1\\-1 & 1\end{bmatrix} = \begin{bmatrix}1 & 1\\-1 & 1\end{bmatrix}\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$.

20. T, by Fact 3.3.10.

21. F; We are unable to find an invertible matrix S as required in the definition of similarity.

22. F; Five vectors in $\mathbb{R}^4$ must be dependent, by Fact 3.2.6.

23. T, by Definition 3.2.1 (all vectors in $\mathbb{R}^3$ are linear combinations of $\vec{e}_1, \vec{e}_2, \vec{e}_3$).

24. T; Use a basis with one vector on the line and the other perpendicular to it.

25. T, since $AB\vec{v} = A\vec{0} = \vec{0}$.

26. T, by Definition 3.2.3.

27. F; Suppose $\vec{v}_2 = 2\vec{v}_1$. Then $T(\vec{v}_2) = 2T(\vec{v}_1) = 2\vec{e}_1$ cannot be $\vec{e}_2$.

28. F; Consider $\vec{u} = \vec{e}_1$, $\vec{v} = 2\vec{e}_1$, and $\vec{w} = \vec{e}_2$.

29. T, since $A^{-1}(AB)A = BA$.

30. T, since both kernels consist of the zero vector alone.

31. T; Consider any basis $\vec{v}_1, \vec{v}_2, \vec{v}_3$ of V. Then $k\vec{v}_1, \vec{v}_2, \vec{v}_3$ is a basis as well, for any nonzero scalar k.

32. F; The identity matrix is similar only to itself.

33. F; Consider $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

34. F; Let $A = I_2$, $B = -I_2$ and $\vec{v} = \vec{e}_1$, for example.

35. F; Let $V = \text{span}(\vec{e}_1)$ and $W = \text{span}(\vec{e}_2)$ in $\mathbb{R}^2$, for example.

36. T; If $A\vec{v} = A\vec{w}$, then $A(\vec{v} - \vec{w}) = \vec{0}$, so that $\vec{v} - \vec{w} = \vec{0}$ and $\vec{v} = \vec{w}$.

37. T; Consider the linear transformation with matrix $[\vec{w}_1 \ \ldots \ \vec{w}_n][\vec{v}_1 \ \ldots \ \vec{v}_n]^{-1}$.

38. F; Suppose A were similar to B. Then $A^4 = I_2$ were similar to $B^4 = -I_2$, by Example 7 of Section 3.4. But this isn't the case: I_2 is similar only to itself.

39. F; Note that $\mathbb{R}^2$ isn't even a subset of $\mathbb{R}^3$. A vector in $\mathbb{R}^2$, with two components, does not belong to $\mathbb{R}^3$.

40. T; If $B = S^{-1}AS$, then $B + 7I_n = S^{-1}(A + 7I_n)S$.

41. T; Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, for example.

42. F; Consider I_n and $2I_n$, for example.

43. T; Matrix $B = S^{-1}AS$ is invertible, being the product of invertible matrices.

44. T; Note that $\text{im}(A)$ is a subspace of $\ker(A)$, so that
$\dim(\text{im } A) = \text{rank}(A) \leq \dim(\ker A) = 10 - \text{rank}(A)$.

45. T; Pick three vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ that span V. Then $V = \text{im}[\vec{v}_1 \ \vec{v}_2 \ \vec{v}_3]$.

46. T; Check that $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ is similar to $\begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$.

47. T; Pick a vector $\vec{v}$ that is neither on the line nor perpendicular to it. Then the matrix of the linear transformation $T(\vec{x}) = R\vec{x}$ with respect to the basis $\vec{v}, R\vec{v}$ is $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, since $R(R\vec{v}) = \vec{v}$.

48. F; If $B = S^{-1}AS$, then $B = (2S)^{-1}A(2S)$ as well.

49. T; Note that $A(B - C) = 0$, so that all the columns of matrix $B - C$ are in the kernel of A. Thus $B - C = 0$ and $B = C$, as claimed.

50. T; Suppose $\vec{v}$ is in both ker(A) and im(A), so that $\vec{v} = A\vec{w}$ for some vector $\vec{w}$. Then $\vec{0} = A\vec{v} = A^2\vec{w} = A\vec{w} = \vec{v}$, as claimed.

51. F; Suppose such a matrix A exists. Then there is a vector $\vec{v}$ in $\mathbb{R}^2$ such that $A^2\vec{v} \neq \vec{0}$ but $A^3\vec{v} = \vec{0}$. As in Exercise 3.4.32a we can show that vectors $\vec{v}, A\vec{v}, A^2\vec{v}$ are linearly independent, a contradiction (we are looking at three vectors in $\mathbb{R}^2$).